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# Financial instruments with credit risk

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## Lecture Summary

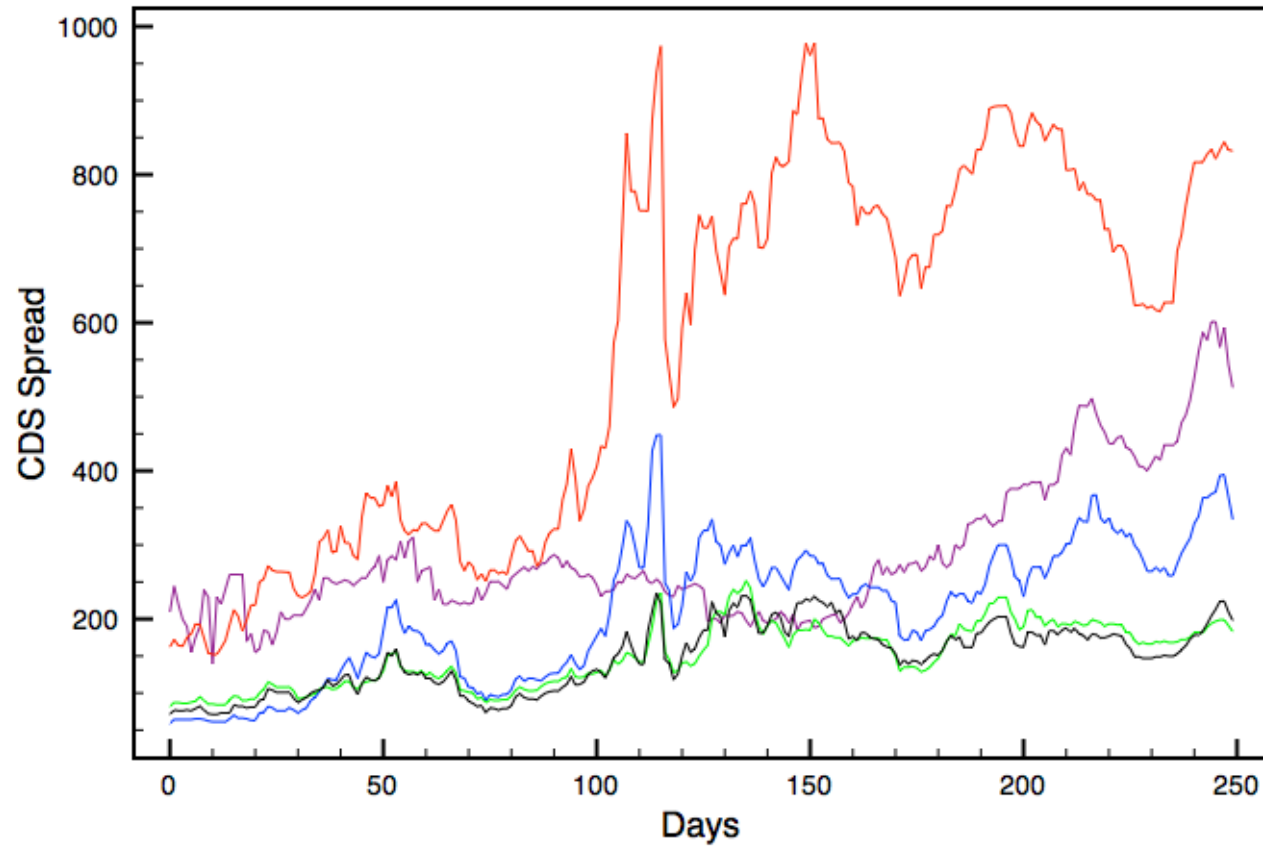
- Example of quoted CDS spreads
- Credit risk hedging using credit-default swaps
- Some historically important recovery ratios
- Corporate bonds and asset swaps
- Exotic single-name credit derivatives
- Portfolio credit derivatives
- Credit-default-swap index
- Major traded indexes

## PIIGS quoted CDS spreads (November 2010)

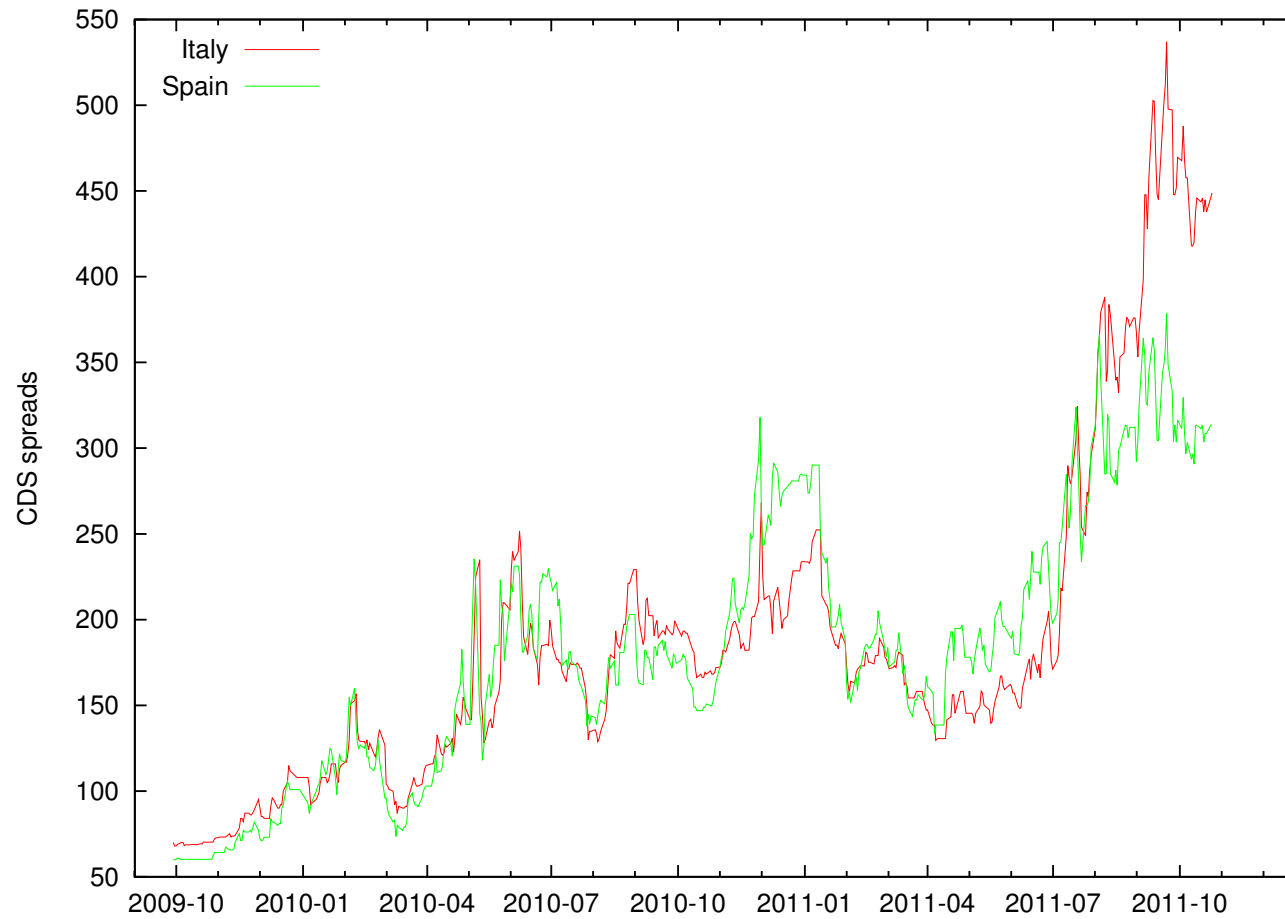
Quoted 5Y CDS spread for PIIGS countries

Date	Portugal	Ireland	Italy	Greece	Spain
15-Nov-10	334.25	512.41	183.05	832.09	197.97
15-Oct-10	279.23	405.14	168.16	625.01	146.90
15-Sep-10	285.21	381.12	199.48	867.71	184.07
16-Aug-10	233.94	294.98	193.39	806.67	175.88
15-Jul-10	246.87	225.00	173.80	747.69	181.36
15-Jun-10	269.14	197.50	197.74	802.04	206.08
17-May-10	263.15	242.50	141.90	640.23	168.00
15-Apr-10	166.79	256.09	125.85	390.54	129.76
15-Mar-10	94.94	250.36	90.05	261.57	77.04
15-Feb-10	183.58	250.30	126.93	318.87	124.96
15-Jan-10	119.79	255.40	107.91	319.81	117.76
15-Dec-09	63.15	179.81	92.06	218.86	81.04

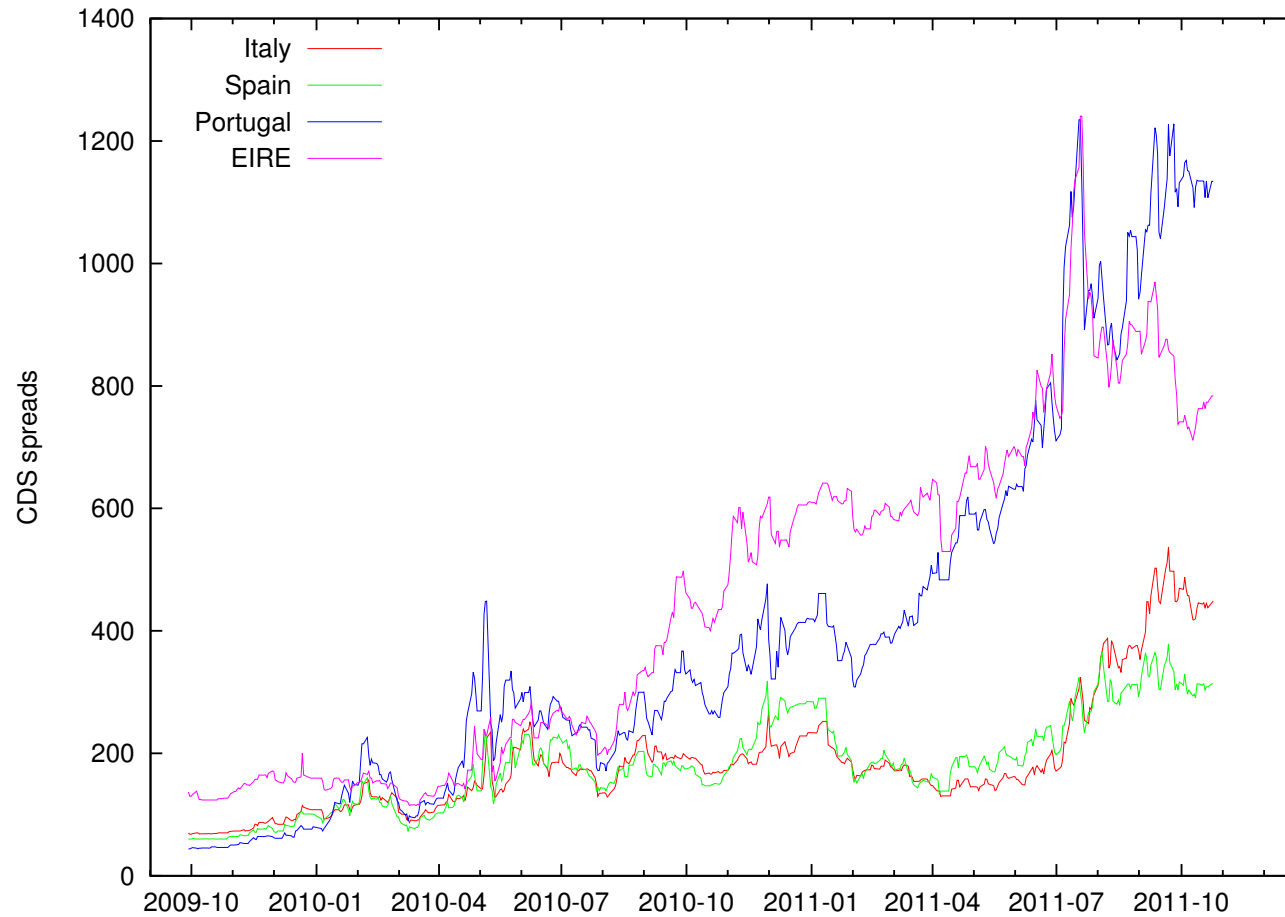
## PIIGS quoted CDS—spread graph November 2010



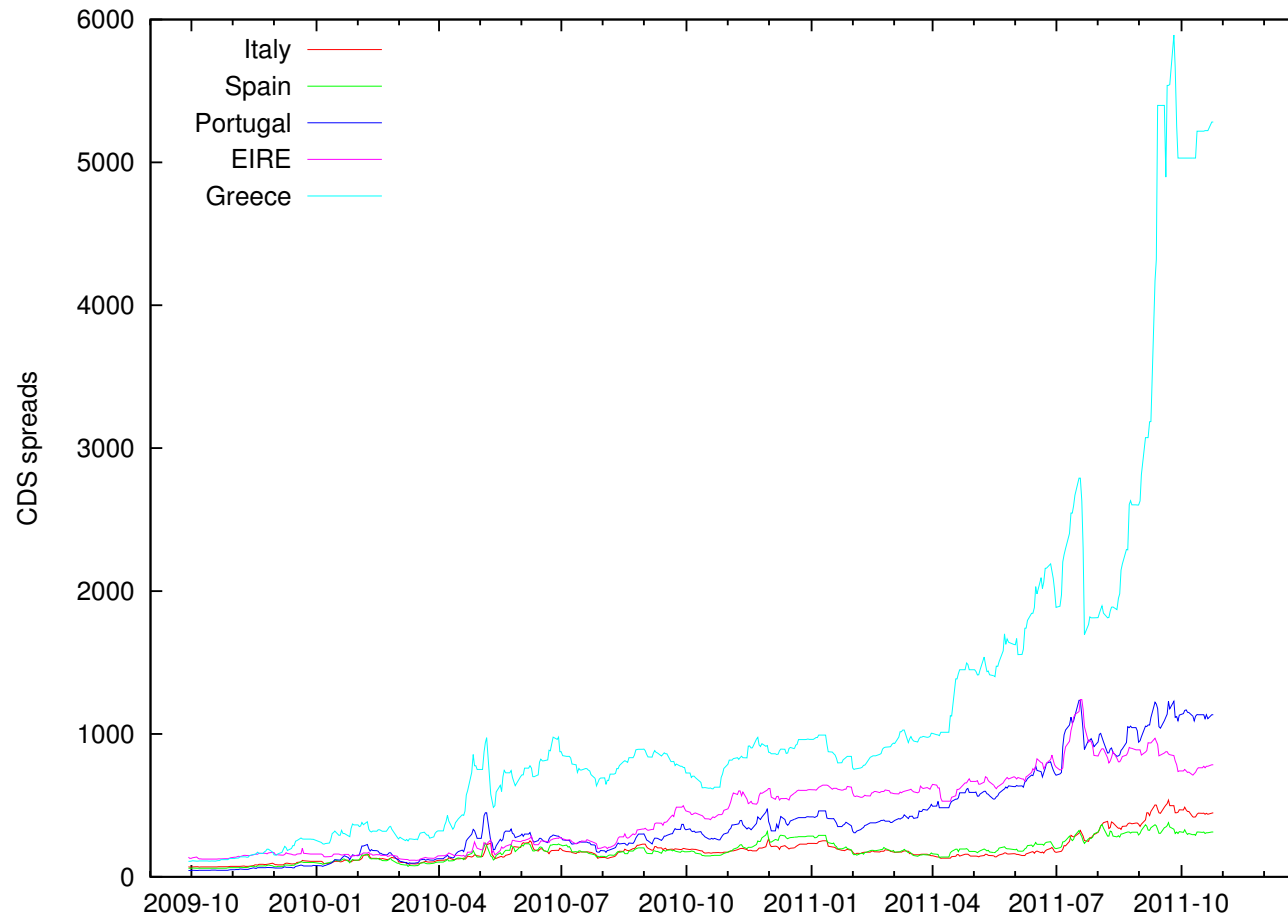
## Italy and Spain—quoted CDS spreads—latest 2 Years



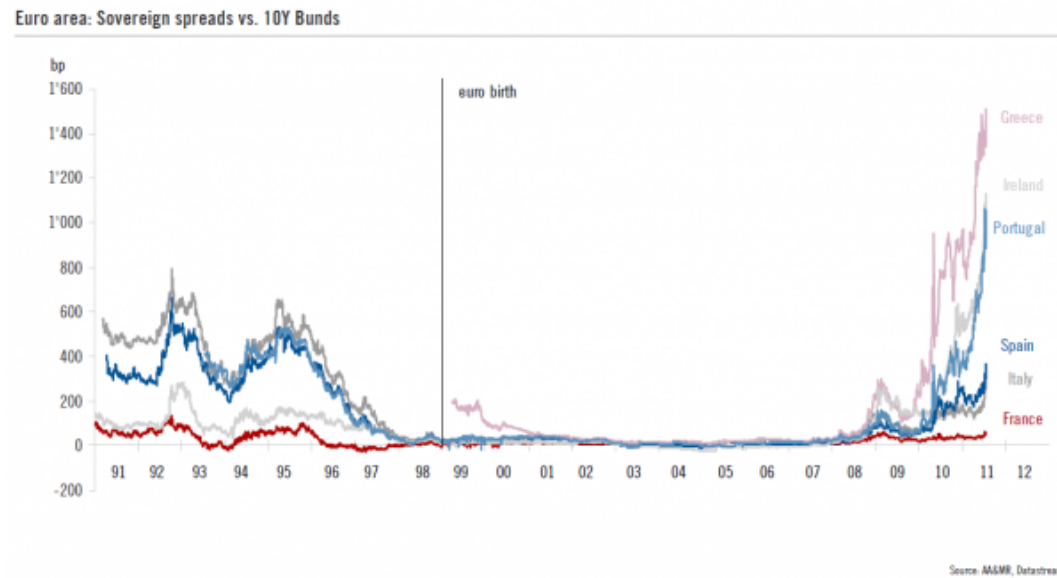
## PIIGS with no G—quoted CDS spreads—latest 2 Years



### PIIGS with G—quoted CDS spreads—latest 2 Years



## Before and after the Euro





## PIIGS default probabilities (November 2010)

Computed default probability for PIIGS countries from quoted 5Y CDS spreads on November, 15th, 2010

<b>Country</b>	<b>Spread</b>	$P_{def}(5Y)$
Portugal	334.25	24.5%
Ireland	512.41	35.0%
Italy	183.05	14.3%
Greece	832.09	50.4%
Spain	197.97	15.4%

## PIIGS default probabilities (October 2011)

Computed default probability for PIIGS countries from quoted 5Y CDS spreads on October, 24th, 2011

<b>Country</b>	<b>Spread</b>	$P_{def}(1Y)$	$P_{def}(5Y)$
Portugal	1134.10	17.49%	61.50%
Ireland	783.52	12.43%	48.27%
Italy	448.84	7.33%	31.48%
Greece	5282.00	59.17%	98.83%
Spain	313.47	5.17%	23.17%

Questions?

## Use of market default probabilities

Default probabilities from CDS quotes are useful to

- Understand the market perception of defaults at future dates
- Compute the fair value of credit-linked financial products
- *Perfectly* hedging a bond issue
- *Partially* hedging credit exposure

## Hedging credit risk with CDSs

Credit default swaps can be used to hedge against the loss incurred in lending money to a certain name

- The CDS hedge will exactly balance the losses (*perfect hedge*) when the losses of a purchased bond listed among the deliverable bonds
- All other losses are only *partially hedged* because, in general, the recovery ratio is not known beforehand

## Some historically important recovery ratios

<b>Date</b>	<b>name</b>	<b>Recovery</b>
2008-10-06	Fannie Mae–Senior	99.90
2008-10-06	Fannie Mae–Subordinated	91.51
2008-10-10	Lehman Brothers	8.625
2008-11-05	Glitnir–Senior	3.000
2008-11-05	Glitnir–Subordinated	0.125
2009-01-14	Republic of Ecuador	31.375
2009-06-12	General Motors CDS	12.50
2009-06-12	General Motors Loans	97.50
2009-11-10	Metro-Goldwyn-Mayer Loans	58.50

Meeting to decide default events: see file  
ISDA-Determinations-Committee-Requests.pdf

Example of recovery auction: see file RR-truvo-21092010.pdf

Questions?



## Bond price in presence of credit risk (1/3)

Problem: price a fixed-rate coupon bond issued by a risky entity: for example a corporate bond when the issuer credit is important

### Input

- quotes for deposits, futures, and swaps
- one or more CDS quotes on the issuer
- the recovery ratio  $R$

### Intermediate

- risk-free term structure for the discount  $D_r(t)$
- term structure for the probability of default  $P(t)$

## Pricing a bond in presence of credit risk (2/3)

Each cash flow must be multiplied by the survival probability at that date and then discounted on on the risk-free term structure  $D_r(t)$ . In case of default the full recovery value is paid back.

For a bond paying coupons  $C_i$ 's the present value is given by

$$\begin{aligned} \text{PV}_{\text{bond}} = & \sum_{i=1}^n [1 - P(t_i)] D_r(t_i) N C_i + [1 - P(t_n)] D_r(t_n) N + \\ & + N R \int_0^{t_n} D_r(s) P(s + ds|s) \end{aligned} \quad (1)$$

The first line is the expected cash flows in case of survival

The second line the cash flows in case of default, i.e.  $RN$ , to be paid at a random time of default

## Pricing a bond in presence of credit risk (3/3)

The last term in (1) can be approximated using the mid-point approximation.

Since  $P(t + dt|t)$  is the default probability between time  $t$  and  $t + dt$

$$\int_0^{t_n} D_r(s) P(s + ds|s) = \quad (2)$$
$$[P(t_1) - P(t_0)] D(t_{1/2}) + [P(t_2) - P(t_1)] D(t_{1+1/2}) + \dots$$

with  $t_1, t_2, \dots$  are future dates on a regular lattice

The dirty price of the bond can be computed from its PV

$$P_{\text{bond}}^{\text{dirty}} = \frac{100 \text{PV}_{\text{bond}}}{N D_r(t_b)}, \quad (3)$$

where  $t_b$  is the bond settlement date

## The asset swap (1/3)

A asset swap allows the holder of a fixed-coupon bond to swap the fixed cash flows from the bond to a stream of cash flows based on the Libor rate plus a spread\*

In swapping the cash flows the investor retains the original credit worthiness of the issuer

Unlike a standard interest-rate swap, the premium is based on the credit exposure to the bond issuer and not on that of the asset-swap issuer.

\*For some reason asset-swap spread is usually denoted by ASW

## The asset swap (2/3)

The asset swap is composed of a fixed leg that pays exactly the same coupons as the original bond, but not the redemption, and a floating leg that pays the Libor rate plus the asset-swap spread  $A$

The floating payment is usually made at a different frequency and accrual basis than the fixed payment. The net present value of the asset swap at the bond settlement date, can be computed as

$$\text{NPV}_{\text{asw}} = N \left( L_{\text{libor}} + A L_{\text{spread}} - L_{\text{fixed}} \right) \quad (4)$$

## The asset swap (3/3)

with

$$L_{\text{libor}} = \sum_{i=1}^{n_{\text{float}}} D_r(\tau_i) L_{i-1} (\tau_i - \tau_{i-1}) - L_0 (t_b - \tau_0) , \quad (5)$$

$$L_{\text{spread}} = \sum_{i=1}^{n_{\text{float}}} D_r(\tau_i) (\tau_i - \tau_{i-1}) - (t_b - \tau_0) , \quad (6)$$

$$L_{\text{fixed}} = \sum_{i=1}^{n_{\text{fixed}}} D_r(t_i) C_i , \quad (7)$$

where

- $\tau_i$  are the payment dates for future floating payments
- $\tau_0$  is the accrual date for the current floating coupon
- $t - s$  is the year fraction between  $s$  and  $t$

## Next coupon for an asset swap

The asset swap can be bought at any time during the life of the bond

When it is bought in between coupon dates the next fixed coupon of the asset swap will completely match that of the bond and no accrual is therefore paid.

## The par asset swap (1/2)

Given the bond price  $P_{\text{bond}}^{\text{dirty}}$ , the asset swap is said to be at par when the portfolio composed by the bond and the asset swap are equivalent to a bond at par, i.e.,

$$P_{\text{bond}}^{\text{dirty}} + 100 \frac{\text{NPV}_{\text{asw}}(t_b)}{N} = 100. \quad (8)$$

The asset swap spread  $A$  can be computed from equations (4–8)

$$A = \frac{1 - P_{\text{bond}}^{\text{dirty}}/100 + L_{\text{fixed}} - L_{\text{libor}}}{L_{\text{spread}}}. \quad (9)$$



## The par asset swap (2/2)

Similarly, given the asset-swap spread for a certain bond, it is possible to compute the implied bond price:

$$\frac{P_{\text{bond}}^{\text{dirty}}}{100} = 1 + L_{\text{fixed}} - L_{\text{libor}} - A L_{\text{spread}}. \quad (10)$$

Therefore, the asset-swap spread and the bond price are equivalent and the knowledge of one is necessary and sufficient to determine the other.

### Cash-flow computation for asset swap

Vodafone Group Plc 6.25% 15Jan2016 EX EUR A- (FIT) Help

XS0408285913=R Default MM MID Price 114.9885 22 Nov10 10:16 TRFIT 114.86 22-Nov-2010

**Bond** BUY

Maturity: 15-Jan-2016  
 Currency: EUR  
 Bond Type: Fixed  
 Coupon: 6.250  
 Frequency: Annual  
 Basis: Actual/Actual  Fair Value in Bond Currency  
 Notional: 100M  
 Market Price: 120,365,212 @25Nov10  
 Accrued: 5,376,712.33

**Swap** for user defined bond ->

Start Date: 25-Nov-2010  
 Currency: EUR  
 Fixed/Floating: Fixed Float  
 Coupon/Spread: 6.250 0.0 bp  
 Frequency: Annual Semi-annual  
 Basis: Actual/Actual MM Act/360  
 Notional: 100,000,000 100,000,000  
 Current Rate: 1.2660 EURIBOR  
 Fair Value EUR: -30,172,442 11,580,314  with accrued  
 Accrued: 0.00 10,550.00  
 Irregular Coupon: None  
 Swap NPV: -18,592,127  
 Total NPV: 101,773,085

**Asset Swap Calculation**

Price: 114.989  
 Yield: 3.052  
 Vanilla Swap: 2.401

Spread

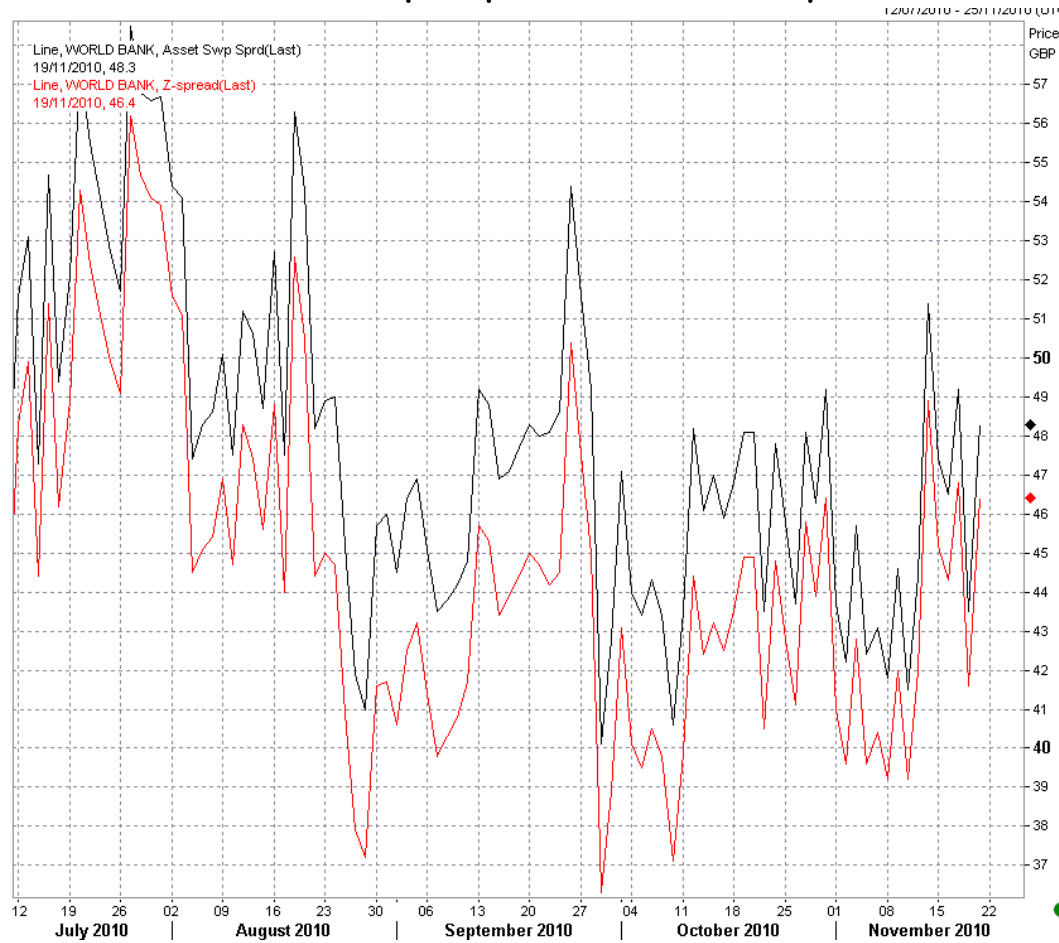
Asset Swap  use clean price: 73.1 bp  
 Upfront Payment: -14,988,500  
 Funding Spread: 0.0 bp

Spread Details	Spread	Money
Upfront	-304.2 bp	-14,988,500.00
Swap	377.34 bp	18,592,127.25
Funding Spread	0 bp	0.00
<b>Total</b>	<b>73.14 bp</b>	<b>3,603,627.25</b>

Adjust Dates and CashFlows to WD

Dates	Bond Cash Flows			Swap Cash Flows			Swap Cash Flows		
	Rate	cashflow	NPV	Rate	cashflow	NPV	Rate	Cashflow	NPV
		Received EUR Fixed 6.25%	124,029,430		Paid EUR Fixed 6.25%	-30,172,442		Received EUR Float 0 bp	11,580,314
17 Jan2011	6.2500	6,250,000	6,242,552	6.2500	-873,288	-872,247	1.2660	186,383	186,161
15 Jul2011							1.3751	683,724	678,272
16 Jan2012	6.2500	6,250,000	6,152,112	6.2500	-6,250,000	-6,152,112	1.5198	781,010	768,778
16 Jul2012							1.9874	1,004,766	979,191
15 Jan2013	6.2500	6,250,000	6,024,576	6.2500	-6,250,000	-6,024,576	2.1661	1,101,093	1,061,379
15 Jul2013							2.2960	1,154,380	1,100,045
15 Jan2014	6.2500	6,250,000	5,880,207	6.2500	-6,250,000	-5,880,207	2.5160	1,285,941	1,209,856
15 Jul2014							2.7710	1,393,219	1,292,775
15 Jan2015	6.2500	6,250,000	5,712,882	6.2500	-6,250,000	-5,712,882	2.9633	1,514,593	1,384,430
15 Jul2015							3.1581	1,587,825	1,428,684
15 Jan2016	6.2500	106,250,000	94,017,101	6.2500	-6,250,000	-5,530,418	3.2962	1,684,708	1,490,743

### Asset-swap spread vs z-spread



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## Credit rating analysis for single names

- Find the quoted credit spreads: bootstrap the probability curve
- Find/Compute the asset-swap spread and the z-spread
- Find/Compute the bond prices and yields
- Compute the basis = CDS - ASW
- Compare risky and riskless yield curves

Vodafone Group Plc 5Y EUR Senior Unsecured
Credit Default Swap

Ticker/RIC: **VOD**
CDS overview ->
CDS index ->
Fenics curve ->
Fenics methodology ->
22 Nov 2010

**CDS Details**

Start Date: 23 Nov 2010 Buy

Tenor & Maturity: 5Y 20 Dec 2015

Frequency & Day Count: Quarterly MM Act/360

Notional & currency: 10m EUR

Seniority & Pay Accrued: Senior Unsecured NO

Recovery Rate & CDS Market Price: 40% 76.45

**Credit Curve Details**

Recovery Rate & Currency: 40% EUR

Curve Source & CDS Source: Reuters EOD Mid Sprd

Skip First Cash Flow & Credit Events: NO MM

**CDS Calculations**

CDS Spread & NPV: 76.45 bp 0.00

DV01 & Accrued: 4,762.33 0.00

**Fundamentals**

Sector: Communications

Ratings: S&P A- (30MAY06), Moody's N/A, Fitch A- (21JUN10)

Use Flat Curve with spread equal 76.45 bp

Maturity	Spreads (Mid)	Def. Prob.
6M	13.68	0.13%
1Y	19.66	0.36%
2Y	35.49	1.24%
3Y	52.77	2.73%
4Y	64.50	4.41%
5Y	76.45	6.49%
7Y	86.12	10.04%
10Y	95.54	15.52%

Upfr SN CDS or Price quoted Indices not supported. Choose another contr., use flat curve or build your own

Deal ID:

Portfolio:

Counterparty: JP Morgan

Description: VOD 20Dec15 10M EUR

Save Deal Setup portfolios here ->

Curve Source: Reuters EOD Track portfolio here ->

**Basis Analysis**

	Bond 1	Bond 2
Basis Analysis	VOD 5.125 10APR15	VOD 6.25 15JAN16
Bond RIC	GB016666734=	GB040828591=
Bond price	109.70	114.98
Bond Yield	2.737	3.055
Asset Swap Spread	55.1 bp	75.5 bp
Z-Spread	53.0 bp	70.8 bp
Basis	CDS - ASW Spread	21.3 bp
		0.9 bp

**Credit Curve** Historical Basis

## Exotic credit-linked notes

See, for example, `Credit.Linked.Note.pdf`

Questions?

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## Portfolio credit derivatives

Portfolio credit derivatives are products that give exposure/protection to the collective credit of a group of names

- Biggest problem of single name CD: low volumes
- Credit risk is usually on a number of names
- Standardized contracts traded on a market (not an OTC)
- Market based means high volumes, smaller bid/ask



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## Credit default swap index

A CDS index is a contract that gives exposure/protection to a standard portfolio of names

Buying a CDS index is like buying a CDS on each index underlying name with a single index spread

- + More liquid than the corresponding CDS portfolio
- Gives a partial hedge to our portfolio

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## Most traded baskets

Historically two big markets developed: Europe and North America

- (Markit) CDX baskets in North America
- (Markit) iTraxx in Europe
- (Markit) iTraxx in Asia
- (Markit) iTraxx SovX for sovereign

Composition of investment-grade iTraxx Europe

see file `itraxx-europe-series-13.pdf`

Composition of investment-grade CDX North America

see file `CDX-IG-14-v1.pdf`

## NPV of CDS index

Neglecting the accrual leg and with no upfront payments we have

$$\text{NPV}_i = N_i [s_i A_i - C_i] \quad (11)$$

for each single name

In the CDS index we assume  $N_i = N/n$  and the same spread  $s$  for all names

$$\text{NPV}_{\text{index}} = \frac{N}{n} \sum_{i=1}^n [s A_i - C_i] \quad (12)$$

## Fair spread of CDS index (1/2)

Is the CDS-index fair spread the average of all single-name spreads?

For each name  $s_i$  is the fair spread, so that

$$s A_i - C_i = (s - s_i) A_i + s_i A_i - C_i = (s - s_i) A_i \quad (13)$$

The CDS-index NPV becomes

$$\text{NPV}_{\text{index}} = \frac{N}{n} \sum_{i=1}^n (s - s_i) A_i = \frac{N}{n} \left[ s \sum_{i=1}^n A_i - \sum_{i=1}^n s_i A_i \right] \quad (14)$$

## Fair spread of CDS index (2/2)

The index fair spread  $\tilde{s}$  is such that  $\text{NPV}_{\text{index}}=0$ ,

$$\tilde{s} = \left[ \sum_{i=1}^n s_i A_i \right] / \left[ \sum_{i=1}^n A_i \right] \quad (15)$$

- Not just the arithmetic average of CDS spreads
- The default legs do not enter this expression

Quotes of major European (corporate) indices as Nov. 19th, 2010

<b>CDS Market Summary</b>				
N.America	Europe	Asia	Sovereigns	Volatility
<b>Credit Indices</b>				
Name	Spread/Price	Unit	Change(%)	
Markit iTraxx Europe	101	Spread	0.76%	
Markit iTraxx Europe Crossover	457	Spread	0.55%	
Markit iTraxx Europe HiVol	151	Spread	-0.87%	
Markit iTraxx Europe Senior Financials	135	Spread	2.27%	
Markit iTraxx Europe Sub Financials	226	Spread	5.66%	



Quotes of major North American (corporate) indices as Nov. 19th,  
2010

<b>CDS Market Summary</b>				
N.America	Europe	Asia	Sovereigns	Volatility
<b>Credit Indices</b>				
Name	Spread/Price	Unit	Change(%)	
Markit CDX.NA.IG	90	Spread	-0.83%	
Markit CDX.NA.IG.HVOL	140	Spread	-0.80%	
Markit CDX.NA.HY	100.79	Price	0.21%	
Markit CDX.NA.HY.B	101.21	Price	0.16%	
Markit CDX.NA.HY.BB	107.11	Price	1.29%	
Markit CDX.NA.XO	208	Spread	3.20%	
Markit CDX.EM	113.06	Price	-0.02%	

## Quotes of major Sovereign indices as Nov. 19th, 2010

### CDS Market Summary

N.America      Europe      Asia      Sovereigns      Volatility

### Sovereign Indices

Name	Spread	Change(%)
Markit iTraxx SovX Western Europe	166	1.14%
Markit iTraxx SovX CEEMEA	203	0.98%
Markit iTraxx SovX Asia Pacific	102	2.34%
Markit iTraxx SovX Global Liquid IG	132*	4.51%
Markit iTraxx SovX G7	73*	7.48%
Markit iTraxx SovX BRIC	103*	13.80%

Quotes of major Sovereign indices as Nov. 19th, 2010

Index	Series	Version	Term	RED Id	On Run	Coupon	Maturity	Date	Time	Comp Price	Theo Price	Comp Spread	Theo Spread	Mid Day Spread Change
CDX.NA.HY	17	1	5Y	2I65BRHM5	Y	500	20Dec16	17Nov11	N1830	89.960%	90.829%	760.02	735.60	
CDX.NA.HY.B	17	1	5Y	2I65BSEN4	Y	500	20Dec16	17Nov11	N1830	92.391%	92.955%	692.70	677.51	
CDX.NA.HY.BB	17	1	5Y	2I65BVBY6	Y	500	20Dec16	17Nov11	N1830	102.061%	102.276%	451.89	446.97	
CDX.EM	16	1	5Y	2I65BZAP7	Y	500	20Dec16	17Nov11	N1830	107.562%	108.050%	331.97	321.74	
CDX.NA.IG	17	1	5Y	2I65BYCN3	Y	100	20Dec16	17Nov11	N1830	98.269%	98.257%	136.46	136.72	
CDX.NA.IG.HVOL	17	1	5Y	2I65B3AS2	Y	100	20Dec16	17Nov11	N1830	93.504%	92.670%	243.00	262.68	
iTraxx Europe	16	1	5Y	2I666VAW5	Y	100	20Dec16	17Nov11	L1600	95.994%	95.751%	187.69	193.22	
iTraxx Europe HiVol	16	1	5Y	2I667LAT3	Y	100	20Dec16	17Nov11	L1600	91.866%	91.841%	285.30	285.92	
iTraxx Europe Crossover	16	1	5Y	2I667KBY3	Y	500	20Dec16	17Nov11	L1600	90.520%	91.088%	760.00	742.84	
iTraxx Japan	16	1	5Y	2I668HBP8	Y	100	20Dec16	18Nov11	T1600	95.507%		194.93		
iTraxx Australia	16	1	5Y	2I668IAP7	Y	100	20Dec16	18Nov11	T1600	95.457%		198.30		
iTraxx Asia ex-Japan IG	16	1	5Y	4ABCAMA17	Y	100	20Dec16	18Nov11	T1600	94.784%		213.57		
iTraxx SDI-75	4	3	10Y	4ABCAIAK1	Y	35	20Jun17	17Nov11			87.873%		291.15	
iTraxx SovX Western Europe	6	1	5Y	5C769MAF8	Y	100	20Dec16	17Nov11	L1600	88.826%	88.858%	357.75	356.93	
iTraxx SovX CEEMEA	6	1	5Y	5C769NAF6	Y	100	20Dec16	17Nov11	L1600	89.236%	89.499%	339.79	333.43	
iTraxx SovX Global Liquid Investment Grade	6	1	5Y	5C769KAF2	Y	100	20Dec16	17Nov11			94.189%		227.10	
iTraxx SovX G7	6	1	5Y	5C769JAF5	Y	100	20Dec16	17Nov11			96.452%		175.99	
iTraxx SovX BRIC	6	1	5Y	4ABCAPAD1	Y	100	20Dec16	17Nov11			95.701%		192.71	
iTraxx SovX Asia Pacific	6	1	5Y	4ABCANAD6	Y	100	20Dec16	18Nov11	T1600	96.398%		177.25		
LCDX.NA	17	1	5Y	5F199GEP8	Y	250	20Dec16	17Nov11	N1600	92.289%		491.13		
iTraxx LevX Senior	6	9	5Y	4ABCJER0	Y	500	20Jun15	18Nov11	L1100	103.000%				
MCDX.NA	17	1	5Y	5A79DPAH0	Y	100	20Dec16	17Nov11	N1830	95.590%		213.75		

markit

## References

1. Wikipedia: [http://en.wikipedia.org/wiki/Credit\\_default\\_swap\\_index](http://en.wikipedia.org/wiki/Credit_default_swap_index)
2. Markit website: <http://www.markit.com/cds>
3. *Options, future, & other derivatives*, John C. Hull, Prentice Hall  
(from sixth edition)