Advanced credit derivatives

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Lecture Summary

- Asset-backed securities
- Simple model for ABS pricing
- Mortgage-backed securities
- Collateralized debt obligations (CDOs)
- Credit derivatives and the 2007-2008 market crisis

Asset Backed Security

An instrument based on the collateralized value of a portfolio of illiquid assets. Recipe for creating an ABS:

- Start from a number of leftover illiquid cash flows (i.e. unsellable by themselves)
- Gather them up, stir them, and make up a fancy name
- Create a company that owns these assets, namely a special-purpose vehicle (SPV), that will become the issuer of the ABS
- Sell the mix hot as one piece or in tranches according to taste

Examples of ABS issues

- Mortgage-backed securities (MBS)
- Home equity loans (HELs, mostly second lien sub-prime mortgages), in fashion from 2004 to 2007
- Auto loans
- Credit card receivables
- Student loans (SLABS)
- Artistic royalties (such as those coming from selling records)

Average-maturity Model for Asset Backed Securities

- Simplified version of an asset-backed security based on stochastic prepayments
- Consider a number m of individuals taking out a loan, for example a credit-card debt, all for the same amount L
- For each period each borrower can decide to either repay the loan in full or to pay an interest r on the amount borrowed
- An SPV collects all the loans and sells them as an ABS
- An investor buys the ABS, from its point of view, with notional $N = m\,L$ where the redemption is paid out at random times

Assumptions of the AMM for ABS

We derive a simple prepayment model useful to compute the net present value of the ABS (e.g. based on credit-card payments)

- Assume a very large, possibly infinitely large, m and a very small loan amount L, possibly infinitesimal, so that $N = m\,L$ is finite
- Also, all the credit risk associated with the possibility of any actor to not be able to pay back the loan will be included in the discount factor

ABS with an unlimited time horizon (1/3)

Loans do not have a maturity date. Initially, at time $t = 0$, we have

$$n(0) = N \tag{1}$$

The amount of notional paid back is proportional to the number of people that, at a certain time, still need to pay back their loan

$$n(t + dt) = n(t) + dn = n(t) - \mu\ n(t)\ dt \tag{2}$$

We can therefore write the differential equation

$$\mathrm{d}n = \mu\ n\ \mathrm{d}t \tag{3}$$

ABS with an unlimited time horizon (2/3)

It is convenient to define the time scale of payments τ

$$\tau = \frac{1}{\mu} \tag{4}$$

Expression (3) becomes

$$\frac{dn}{dt} = -\frac{n}{\tau}. \tag{5}$$

The only solution satisfying the initial condition (1) is

$$n(t) = N\, e^{-\frac{t}{\tau}}. \tag{6}$$

ABS with an unlimited time horizon (3/3)

It can be shown that τ is the average time of repayment:

$$\begin{aligned}
\bar{t} &= \int_0^\infty t \; \frac{1}{N} \; (-dn) \\
&= \int_0^\infty t \; \frac{1}{N} \; \frac{n}{\tau} \; dt \\
&= \int_0^\infty t \; \frac{1}{N} \; \frac{1}{\tau} N \; e^{-\frac{t}{\tau}} \; dt \\
&= \int_0^\infty \; \frac{t}{\tau} \; e^{-\frac{t}{\tau}} \; \frac{dt}{\tau} \; \tau \\
&= \; \tau \int_0^\infty \; \xi \; e^{-\xi} \; d\xi = \tau
\end{aligned}$$

Finite maturity horizon (1/3)

Consider the case where each individual needs to repay his loan before a certain maturity T.

The proportionality constant μ should increase as time t approaches to T, we define

$$\mu = \frac{1}{\tau} \frac{1}{1 - \frac{t}{T}} . \tag{7}$$

The parameter $\frac{1}{\tau}$ gives the right time scale to μ

Finite maturity horizon (2/3)

For small times compared with T we obtain the unlimited time horizon

$$\lim_{t \to 0} \mu = \frac{1}{\tau}, \tag{8}$$

The denominator of μ, $1 - \dfrac{t}{T}$, is such that

$$\lim_{t \to T^-} \mu = +\infty \tag{9}$$

so that all the prepayments will be made before time T with probability one.

Finite maturity horizon (3/3)

The differential equation associated to $n(T)$ is therefore

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\frac{1}{\tau}\,\frac{1}{1-\dfrac{t}{T}}\;n. \tag{10}$$

This equation can be solved to give

$$n(t) = \;N\left(1-\frac{t}{T}\right)^{\dfrac{T}{\tau}}. \tag{11}$$

The average time of pre-payments can be computed as,

$$\bar{t} = \frac{\tau}{1+\dfrac{\tau}{T}} \tag{12}$$

Summary of ABS models

We derived a parametric form for the outstanding notional as a function of time.

In the case of unlimited time horizon

$$n(t) = N\ e^{-\frac{t}{\tau}} \tag{13}$$

where τ is the expected time of prepayment

When all the prepayments have to be made before a time T, the distribution of outstanding notionals can be model as

$$n(t) = \ N\left(1 - \frac{t}{T}\right)^{\frac{T}{\tau}} \tag{14}$$

Questions?

Mortgage Backed Securities

An instrument based on the value of a portfolio of (guaranteed) home loans

- A number of similar mortgages (similar by maturity, region, and payment structure) gathered together to form a *mortgage pool*
- The MBS issuers gathers all the payments from the mortgages and passes them to the MBS holders (pass-through MBS)
- The MBS issuers usually gives additional guarantees on the payments (who guarantees for the MBS issuers?)

Actors in a MBS

- The *mortgagor* or *mortgager*: the borrower in a mortgage agreement
- The *mortgagee*: the lender or creditor in a mortgage agreement
- The *issuer*: bundles the mortgages of one or more mortgagees into an MBS
- The *investor*: buys the MBS in the hope to make a good return

Risks of MBS

- Interest-rate risk: fixed-rate payments can be less appealing when interest-rates rise
- Credit risk: some home-owners may not be able to pay their mortgage: the house is repossessed and sold. If the MBS guarantees, it may not have enough cash to pay
- Pre-payment risk: if interest-rates go down, some home owners may decide to re-finance their mortgage at a lower rate, i.e., prepay their mortgage

Pre-payment risk of MBS

- Many mortgages feature full prepayment flexibility (US first, Denmark, other European countries more recently)
- The mortgagor is long a pre-payment option that can exercise if she finds a better mortgage
- An investor in an MBS is short a prepayment option
- Careful modeling of homeowners prepayment behavior is crucial for valuation and risk management of MBS.

Pricing MBS

Approaches to MBS pricing

- Pure *macro-economic* model. All pre-payments depend on macro-economic factors (employment, GDP changes, ...). Model turnover.
- Pure *risk-neutral* model. All pre-payments are rational decisions about interest-rate movements. The mortgagor owns an amortizing swaption
- Mixed model. A reasonable mix of macro-economic and option-theoretic models

Purely macro-economic model to MBS pricing

- All pre-payments depend on macro-economic factors (employment, GDP changes, ...). Model turnover.
- Surveys and macro-economic data are used to **estimate** the model parameters
- These models do **not** provide arbitrage-free prices

Mortgage turnover

The mortgage turnover v is the average number of pre-payments in one year. E.g., $v = 6\%$, the number of investors in the next year is:

$$n(t+1) = n(t)(1-v) \tag{15}$$

Assuming the exponential decay, $n(t) = N\ e^{-t/\tau}$, we have

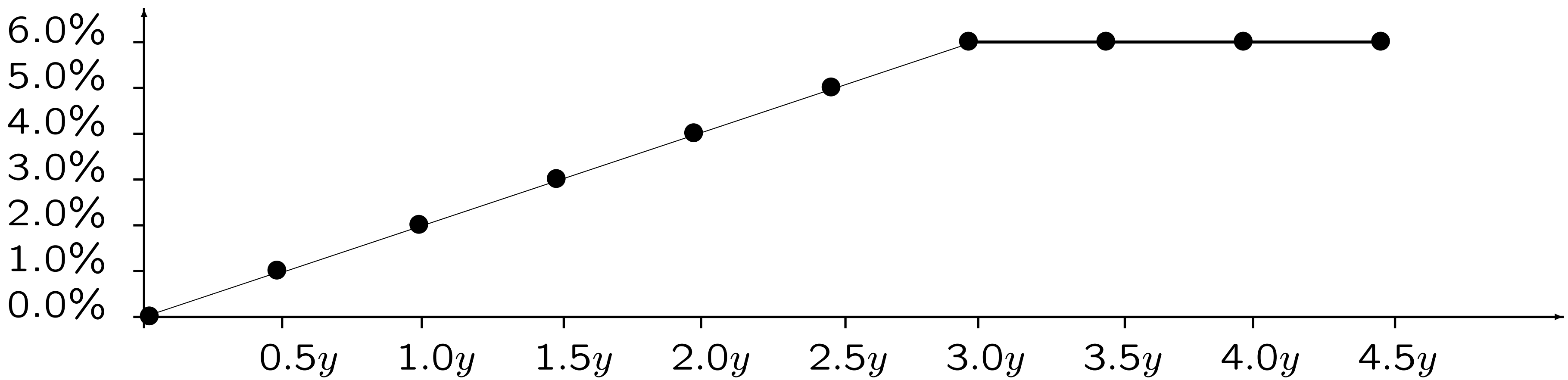
$$1 - v = \frac{n(t+1)}{n(t)} = \frac{e^{-(t+1)/\tau}}{e^{-t/\tau}} = e^{-1/\tau} \tag{16}$$

so that we obtain a relationship between v and τ

$$v = 1 - e^{-1/\tau} \qquad \Longleftrightarrow \qquad \tau = \frac{1}{\log\left[\frac{1}{(1-v)}\right]} \tag{17}$$

Time-dependent mortgage turnover

The turnover depends on time: e.g., from 0% to 6% in three years



After the third year the turnover is constant. Compounding is usually yearly or twice a year

Purely risk-neutral models to MBS pricing

- All pre-payments are rational decisions about interest-rate movements and interest-rate volatilities
- The option to prepay the mortgage is an amortizing American swaption
- The mortgagor is a quant and can evaluate the swaption
- The mortgagor can determine when it is optimal to exercise the swaption

Amortizing swaption payoff (1/2)

An amortizing swaption is an option on an amortizing forward swap

$$\text{Swap}_K = A_{\text{Libor}} - r_K\, A_{\text{Fixed}} \tag{18}$$

where A_{Libor} is the PV of an amortizing Libor leg and

$$A_{\text{Fixed}} = n(d_1)D(d_2)\,T(d_1, d_2) + \ldots + n(d_{m-1})D(d_m)\,T(d_{m-1}, d_m)$$

To be compared with the "fair" amortizing forward swap

$$\text{Swap}_{\text{fair}} = A_{\text{Libor}} - r_{\text{fair}}\, A_{\text{Fixed}} \tag{19}$$

with $r_{\text{fair}} = A_{\text{Libor}}/A_{\text{Fixed}}$

Amortizing swaption payoff (2/2)

Since the payoff is cash settled we have

$$\begin{aligned} \text{Payoff} &= \max\left(\text{Swap}_K - \text{Swap}_{\text{fair}}, 0\right) \\ &= \max\left(r_{\text{fair}}\, A_{\text{Fixed}} - r_K\, A_{\text{Fixed}}, 0\right) \\ &= A_{\text{Fixed}} \max\left(r_{\text{fair}} - r_K, 0\right) \qquad (20) \end{aligned}$$

remember $A_{\text{Fixed}} > 0$.

We **cannot** assume r_{fair} to be log-normal. Hence we cannot use the Black formula (not consisten with log-normal quoted IRS swaptions).

To value the swaption we need an interest-rate model such as Hull-White or the swaption-market model

Option theoretic approach to price MBS

Consider the mortgage from the point of view of the home owner

- Compute the PV of the mortgage cash flows using a risky curve
- Compute the price of the amortizing swaption using an interest-rate model

Value the MBS as a mortgage which is callable and amortizing, however, secured by the house value

Mixed model approach to MBS pricing

- A reasonable mix of macro-economic and option-theoretic models

Kalotay-Fabozzi model for MBS

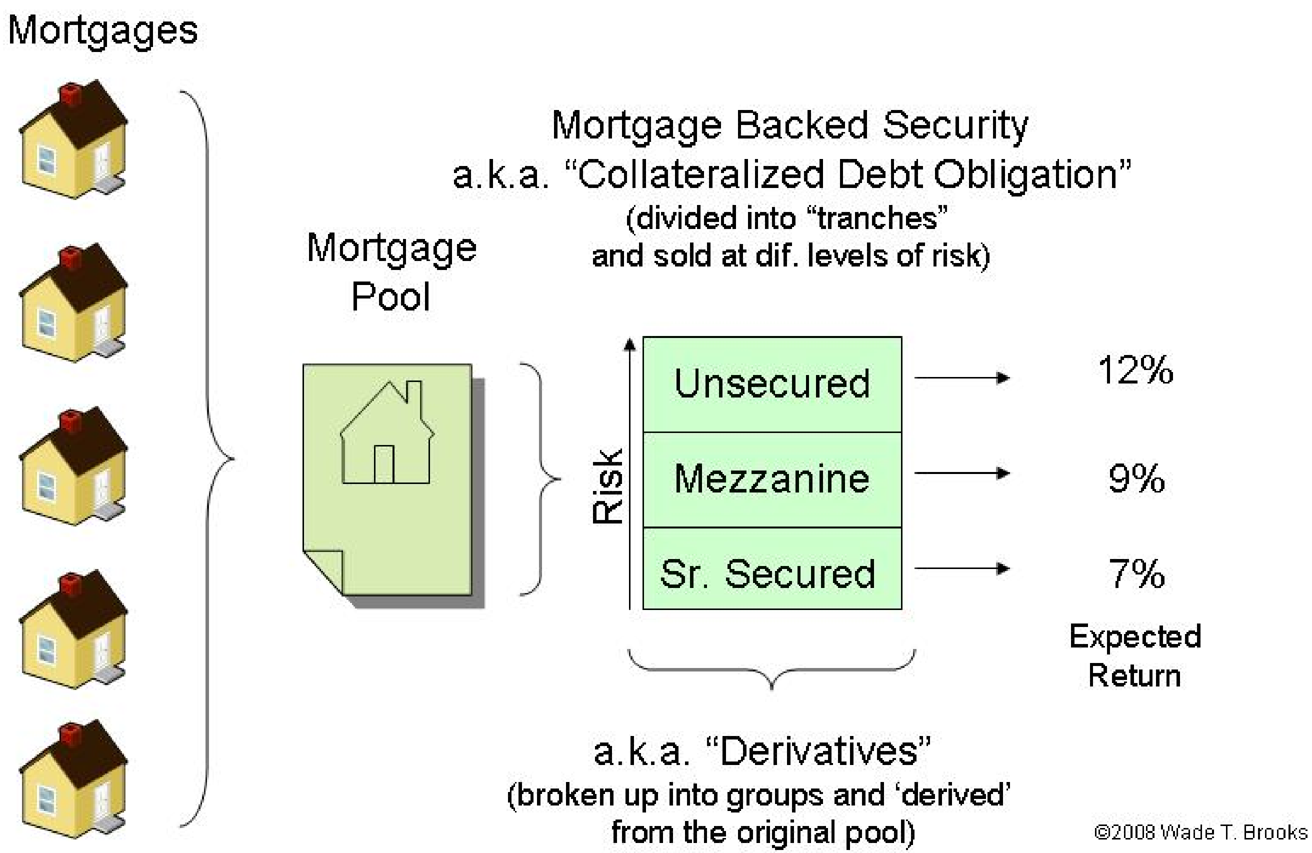
- Consider "deterministic" pre-payments like an ABS
- Consider each mortgage using an option-theoretic approach (amortizing swaption)
- Collect mortgages in different pools of prepayments
- Discount is made using the MBS-issuer risky curve

Questions?

Collateralized Mortgage Backed Securities

In the CMBS the same Mortgage pool is divided up in tranches

- The unsecured tranche has the highest risk and return
- The mezzanine tranche are touched only when the unsecured tranche is exhausted
- The senior tranche is the last one to be touched


Mortgages
Mortgage Backed Security
a.k.a. "Collateralized Debt Obligation"
(divided into "tranches"
and sold at dif. levels of risk)
Mortgage
Pool
Risk
Unsecured
Mezzanine
Sr. Secured
12%
9%
7%
Expected
Return
a.k.a. "Derivatives"
(broken up into groups and 'derived'
from the original pool)
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Collateralized debt obligations

CDOs are the shooting stars of credit markets

According to Wikipedia the global CDO issuance in billion dollars was

2004	157.4
2005	251.3
2006	520.6
2007	481.6
2008	61.9
2009	4.3
2010	8.0

Bond portfolios sold in tranches

Consider the following bond portfolio

Issuer	**Notional** (M$)	**Coupon**	**CDS Spread**	P_{def}(5Y)
Gallo porto	2,000	4.20%	334.25	24.5%
Stout State	2,000	7.25%	512.41	35.0%
Poveri Talia	2,000	2.45%	183.05	14.3%
Souvlakia	2,000	8.50%	832.09	50.4%
Ivana Sp.a	2,000	2.60%	197.97	15.4%
Portfolio	10,000	5%	425	37%

Bond portfolios sold in equal tranches

Suppose the bond portfolio is sold to four parties equally

Tranche	**Def. Risk** (K$)	**Coupon** (K$)	**Coupon** (%)
Tranche 1	2,500	125	5.00%
Tranche 2	2,500	125	5.00%
Tranche 3	2,500	125	5.00%
Tranche 4	2,500	125	5.00%
Portfolio	10,000	500	5%

Like in a mutual fund all the participants have the same gain and bear the same risk

Bond portfolios sold in CDO tranches

The tranches are not sold equally

Tranche	**Def. Risk** (K$)	**Coupon** (K$)	**Coupon** (%)
Equity	(1^{st}) 1,250	125	10.00%
Junior	(2^{nd}) 1,750	125	7.143%
Senior	(3^{rd}) 3,000	125	4.167%
Super sen.	(4^{th}) 4,000	125	3.125%
Portfolio	10,000	500	5%

- Junior tranches have higher coupons and bear more risk
- Senior tranches have lower coupons and bear less risk

CDOs of CDSs

To decrease credit risk and to increase the leverage of bond CDOs people started to create CDOs on portfolios of CDSs:

The risk of a portfolio of CDSs is sold in tranches to investors with different risk appetite

The riskiest tranche, namely the *equity* tranche, was usually kept by the structurer

To increases leverage further pretty soon tranches were sold without an underlying portfolios of CDSs

Single tranche CDOs

The single-tranche collateralized debt obligation is an extension of a credit default swap to the losses generated by a basket of underlying names

Given the basket notional a precise loss tranche is singled out: for example all the losses incurred between 6%, the attachment point, and 9%, the detachment point

The protection buyer pays a spread on the tranche notional so that he can receive a reimbursement for all the losses incurred in the portfolio after the attachment point and up to the detachment point

Quoted single-tranche CDOs

Single-tranche CDOs have been standardized and are traded since 2004 on two major indexes: the European iTraxx and the North American CDX

While for credit default swaps the most important factor in pricing is given by the expected probability of default of the underlying name, the value of a CDO tranche is also dependent on the correlation of more than one default to occur in a period of time

Single-tranche CDOs are (were) quoted in terms of correlation assuming the default probability from quoted CDS indexes

Questions?

Correlation crisis of 2005

After S&P downgraded the car makers GM and Ford on May, 5th, 2005, a sharp rise in the CDS spread of most names followed

As a consequence the implied CDO correlations soared, in turn, the relative price of different tranches spiked. The resulting CDO prices went out of the traders comprehension and panic ensued

Eventually, the CDS spreads return to previous level but the implied correlation never did

In between crisis: 2006

The correlation crisis of 2005 mostly happened because people did not understand the limits of the model they were using for pricing single-tranche CDOs

It was an early sign of what was going to happen on a larger scale few years later

Unfortunately, pretty soon after May 2005 large investment banks continued to make money with little-understood credit derivatives

Some causes of the crisis of 2007-2008

People frustrated by low interest rates (2% – 4%) bought anything that had an higher return (5% – 8%)

However they did not know what they bought

Because the sellers did not know it themselves

Many instruments were totally not standard OTC transactions with very high liquidity risk

Inadequate instruments were used to compute liquidity risk

Remarks by Chairman Alan Greenspan

On May 5th, 2005, FED chairman Alan Greenspan said:
The Federal Reserve remains concerned that the stress tests that some large participants are using to evaluate potential losses in the event of a large participant's default do not fully capture the potential interaction of counter-party credit risk and market risk, especially in concentrated markets.

- Banks initially used CDS to decrease their credit risk
- Eventually (around 2004) they sold them to make *"free"* money

Warren Buffet on Derivatives (1/2)

Following are edited excerpts from the Berkshire Hathaway annual report for 2002.

I view derivatives as time bombs, both for the parties that deal in them and the economic system. Basically these instruments call for money to change hands at some future date, with the amount to be determined by one or more reference items, such as interest rates, stock prices, or currency values. For example, if you are either long or short an S&P 500 futures contract, you are a party to a very simple derivatives transaction, with your gain or loss derived from movements in the index. Derivatives contracts are of varying duration, running sometimes to 20 or more years, and their value is often tied to several variables.

Warren Buffet on Derivatives (2/2)

Unless derivatives contracts are collateralized or guaranteed, their ultimate value also depends on the creditworthiness of the counter-parties to them. But before a contract is settled, the counter-parties record profits and losses often huge in amount in their current earnings statements without so much as a penny changing hands. Reported earnings on derivatives are often wildly overstated. That's because todays earnings are in a significant way based on estimates whose inaccuracy may not be exposed for many years.

Important historical moment of the financial crisis

June 2003	Feds lower the rate to 1%. People look for alternative investments
May 2005	Early warning: correlation crisis
Aug. 2006	U.S. Home Construction Index is down over 40%
Feb. 2007	Sub-prime industry collapse Several sub-prime lenders declaring bankruptcy
2007–2008	Collapse of many financial players
2008	In September Lehman Brothers is **not** bailed out
2010-2011	Crisis spreads to sovereign states
2012	Default of an European sovereign state

References

1. Wikipedia: `http://en.wikipedia.org/wiki/Asset-backed_security`

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3. *Options, future, & other derivatives*, John C. Hull, Prentice Hall (from fourth edition)

