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# Risk simulations for a bond in QuantLibXL

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## StatPro asset coverage

- Equities, equity indexes, and funds
- Equity options: vanilla options, digital and barrier options, ...
- Interest-rate products: IR swaps, IR caps and floors, swaptions, constant-maturity swaps, overnight swaps, ...
- Bonds: fixed-rate, floating-rate, reverse floaters, CMS bonds, ...
- Equity-linked bonds: bonds with embedded options, Himalaya, ...
- Inflation-linked bonds, inflation derivatives
- Foreign-exchange instruments: cross-currency swaps, options, ...
- Credit derivatives: credit-default swaps, ...
- Mortgage-backed securities, asset-backed securities, ...
- ...

## Executive summary

- Consider a simple non-trivial instrument: the fixed-rate coupon bond
- Consider in details the yield-curve and credit-spread risk factors
- Show from the beginning to the end the computation of risk figures using tools available in QuantLib

## Talk outline

- Part I: QuantLib tools for pricing (buy side)
- Part II: QuantLib tools for risk management

Examples using the QuantLib Addin and QuantLibXL (ver. 1.0.1)

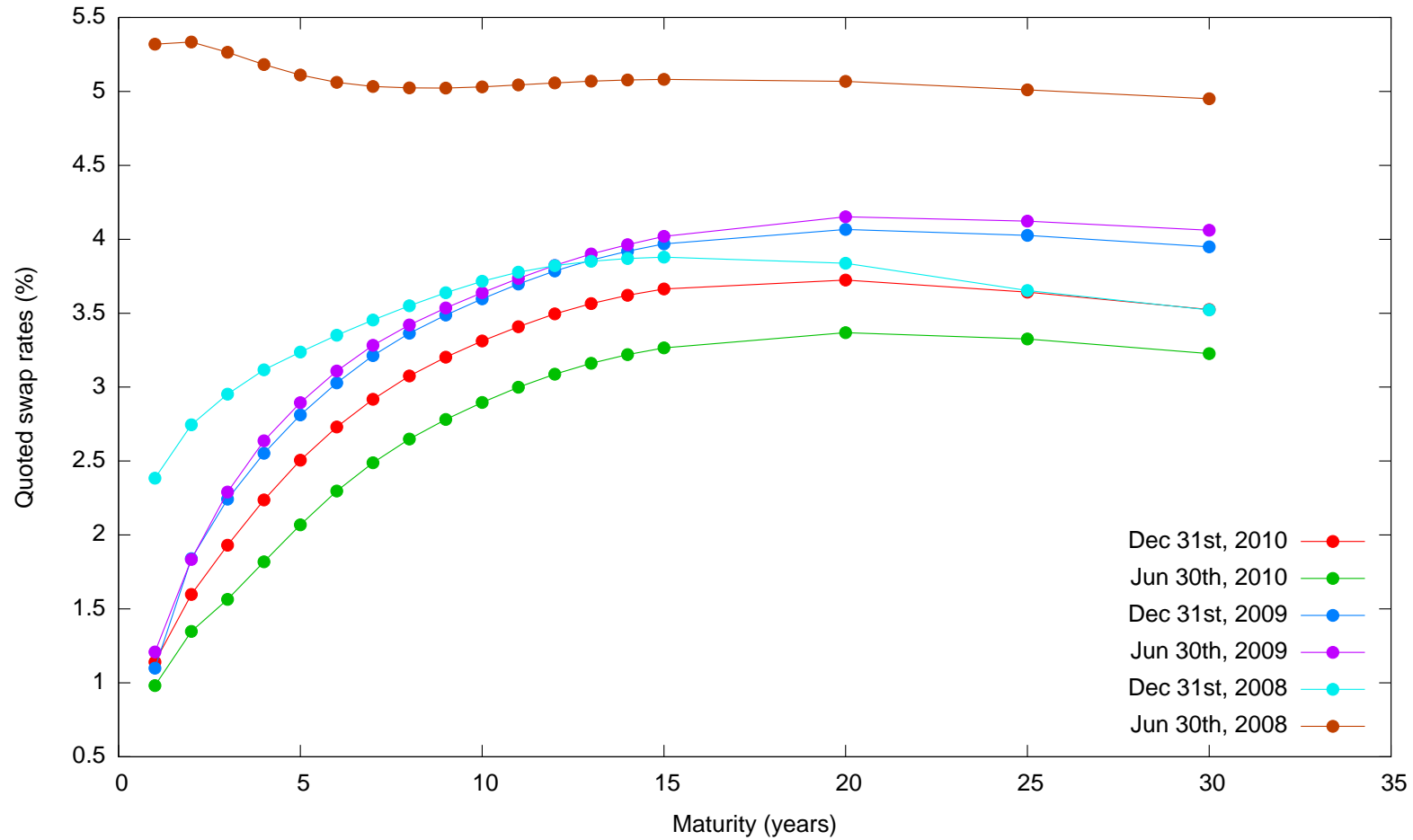
## Part I: Pricing tools

1. Bootstrap of a yield curve from quoted swap rates
2. Creation of a spreaded curve from any yield curve
3. Computation of price for a fixed-rate coupon bond
4. Examples using the QuantLib Addin and QuantLibXL

## Interest rate curve

- Building block of financial engineering
- Used to compute the present value of cash flows paid at future known dates
- Should be calibrated from market quotes
- We assume the risk-free curve to be calibrated using swap rates

### Historical swap rates (Eurirs)



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## Bootstrap of a yield curve: many choices

- Choose the financial instruments that will make the curve up: quoted fair swap rates (otherwise deposit rates, IR futures, ...)
- Choose a financial quantity to interpolate: discount rate (otherwise zero rate, forward rate, ...)
- Choose an interpolator: log-linear (otherwise linear, forward flat, backward flat, ...)



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## Bootstrap of a swap curve in the QuantLib Addin

Create swap-rate helpers for the bootstrap

```
qlSwapRateHelper2("swap-10y",  
    Rate, Tenor, Calendar, FixedLegFrequency,  
    FixedLegConvention, FixedLegDayCounter, IborIndex,  
    Spread, ForwardStart, DiscountingCurve) -> "swap-10y#0000"
```

Create and bootstrap the curve

```
qlPiecewiseYieldCurve("swap-YTS", NDays,  
    Calendar, RateHelpers, DayCounter,  
    Jumps, JumpDates, Accuracy, TraitsID,  
    InterpolatorID) -> "swap-YTS#0000"
```

## Discount curve as a spreaded swap curve

Given the yield curve we can compute the “*risk-free*” discount at a future date  $T$

$$D_{\mathcal{S}}(T) = D(T; S_{1Y}, \dots, S_{30Y})$$

The “*risky*” discount can be computed applying an extra credit spread  $z$  to the swap curve

$$D_{\mathcal{S}}^z(T) = e^{-zT} D_{\mathcal{S}}(T)$$

## Spreaded discount curve using the QuantLib addin

Computing the discount factor using the QuantLib addin

```
qlYieldTSDiscount("swap-YTS", DfDates,  
                  AllowExtrapolation) -> 0.981536
```

Using the QuantLib addin to create a spreaded discount curve

```
qlForwardSpreadedTermStructure("discount-YTS"  
                                "swap-YTS", zspread)  
-> "discount-YTS#0000"
```

## Credit risk modeled with credit spread

Market participants assign a penalty to credit risk.

We assume that

- Quoted financial instruments are at equilibrium for credit risk
- For each bond a single credit spread represents the risk-neutral expectation of credit risk
- Prices are obtained discounting cash flows on spreaded risk-free yield curves

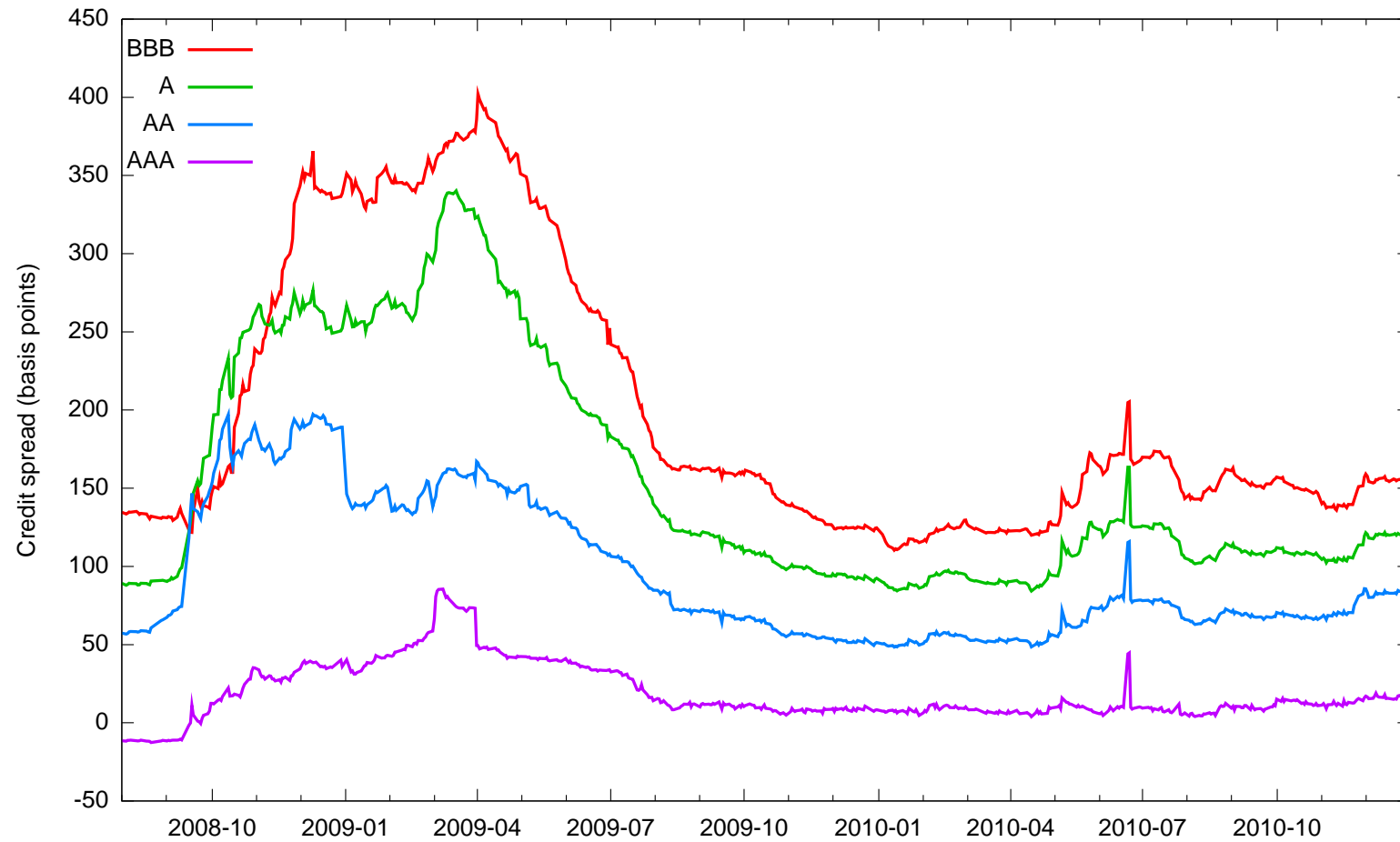
## Pricing fixed-rate coupon bond

The price of a fixed-rate coupon bond can be computed discounting future coupons on a risky discount curve

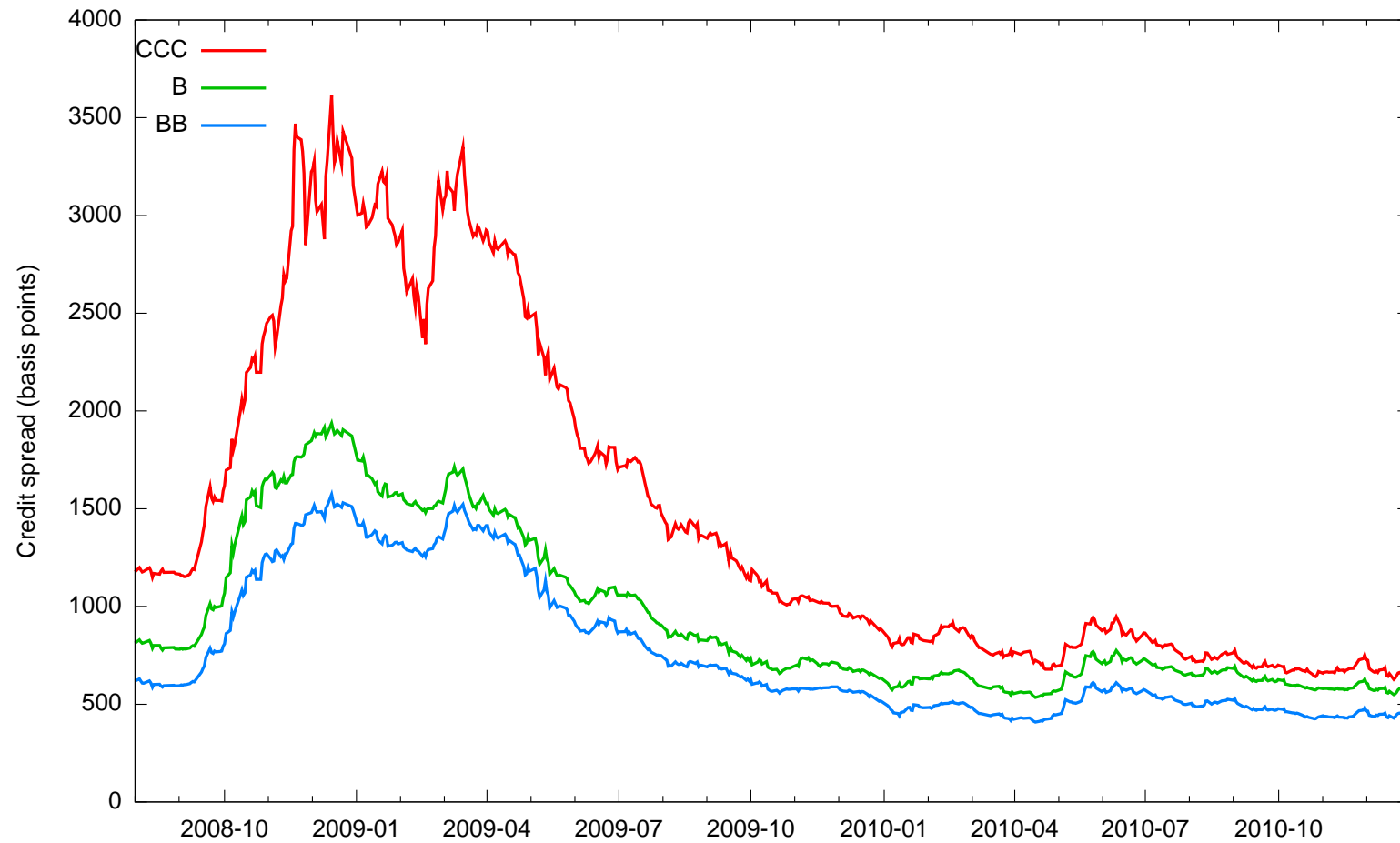
$$\begin{aligned} P(\mathcal{S}, z) &= C_1 e^{-zT_1} D_{\mathcal{S}}(T_1) + \dots + [C_n + R] e^{-zT_n} D_{\mathcal{S}}(T_n) \\ &= C_1 D_{\mathcal{S}}^z(T_1) + \dots + [C_n + R] D_{\mathcal{S}}^z(T_n) \end{aligned}$$

- Assume the credit spread  $z$  to be what is needed to match quoted bond prices
- The credit spread can be observed for bonds quoted for issuers of different sectors, currencies, and ratings

## Historical investment-grade credit spreads



## Historical high-yield credit spreads



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## Pricing fixed-rate coupon bond in QuantLib addin

Create the fixed-rate coupon bond and its engine

```
bond = qlFixedRateBond("bond", Description, Currency,  
                        SettlementDays, FaceAmount, ScheduleID,  
                        Coupons, DayCounter, PaymentBDC,  
                        Redemption, IssueDate, PaymentCalendar, ...)
```

```
engine = qlBondEngine("bond-engine", "discount-curve")
```

Set the engine and compute the price

```
qlInstrumentSetPricingEngine("bond", "bond-engine")
```

```
clean-price = qlBondCleanPrice("bond")
```



## Part II: Risk simulations

1. Simulations of swap curves
2. Static simulations of credit spreads
3. Dynamic simulations of credit spreads
4. Simulations of bond prices
5. Computation of risk figures

## Risk-management simulations

Given a bond we would like to come up with realistic scenarios for its price at a future date (e.g. tomorrow)

- We consider the financial factors affecting the price: the risk factors
- We create simulation scenarios for each risk factor and compute the bond price assuming the realization of each scenario
- The array of computed prices can be used to compute risk-management quantities such as *value at risk*

## Historical simulations of swap curves

We do not know what the swap rates will be tomorrow. However, based on the recent past, e.g. two years, we can simulate a number of possible scenarios for tomorrow's swap rates.

Assumptions:

- Tomorrow's swap rates will be similar to today's rates
- The (absolute) variations of swap rates observed historically from day to day are applied to current swap rates to create the scenarios
- In each scenario assume the swap rates as simulated and bootstrap the interest-rate curve to obtain a discount curve

## Historical scenarios for swap rates

For each historical date  $i$  consider the increment in the swap rates with respect to the previous day

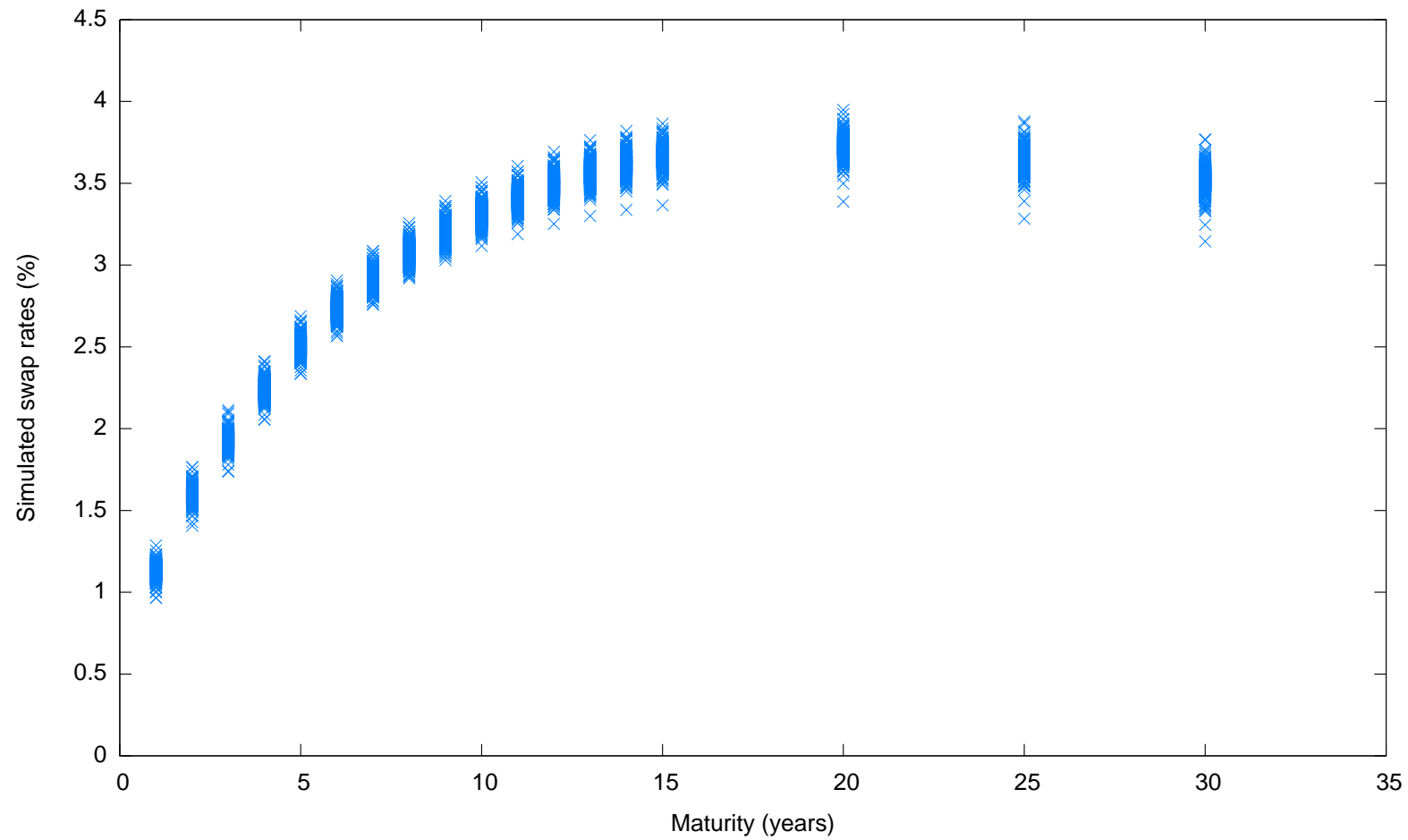
$$\Delta S_{1Y}^i = S_{1Y}^i - S_{1Y}^{i-1} \quad \dots \quad \Delta S_{30Y}^i = S_{30Y}^i - S_{30Y}^{i-1}$$

To compute the swap scenarios, apply these increments to the current rates ( $i=0$ )

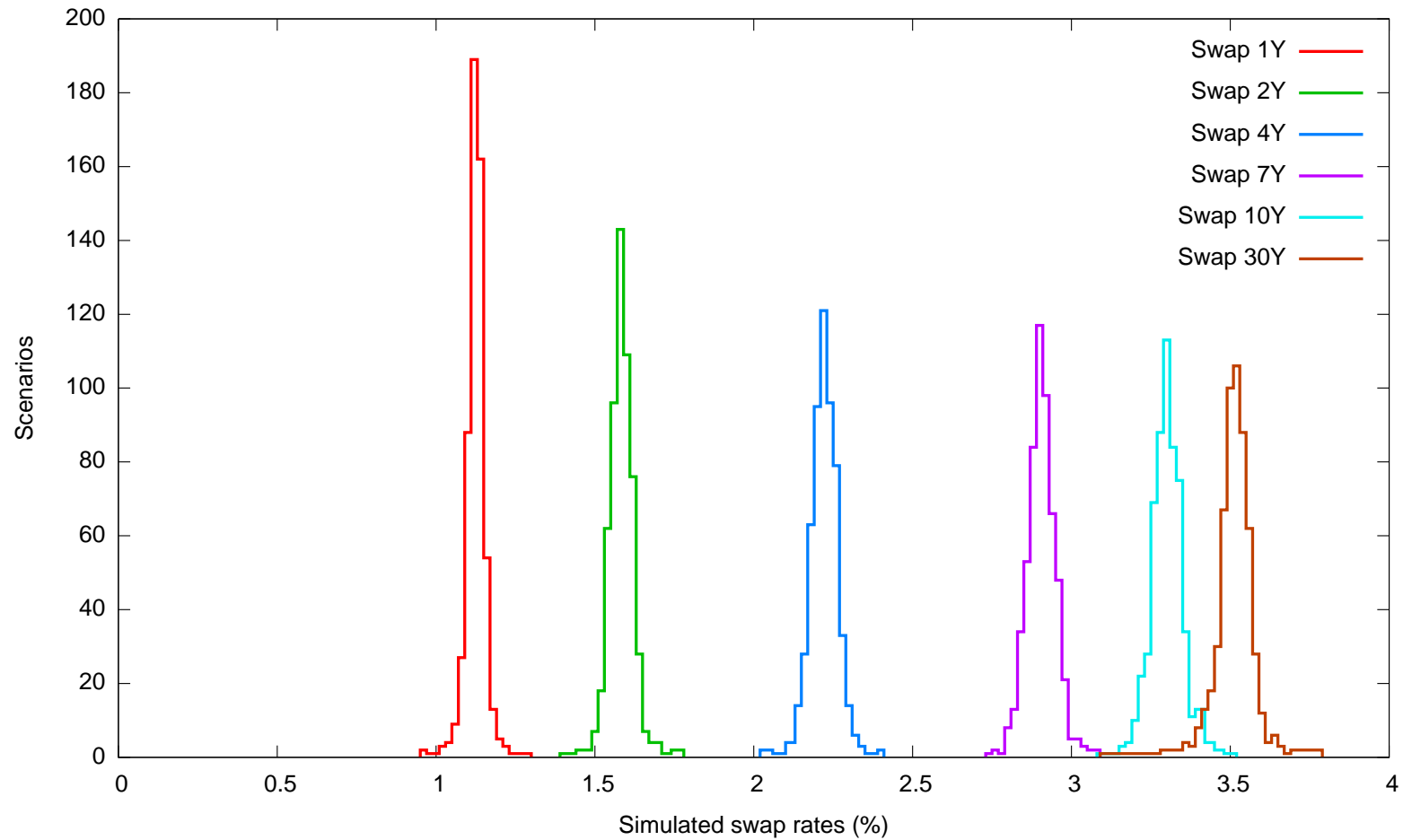
$$S_{\text{sim}(1Y)}^i = S_{1Y}^0 + \Delta S_{1Y}^i \quad \dots \quad S_{\text{sim}(30Y)}^i = S_{30Y}^0 + \Delta S_{30Y}^i$$

Note: none of the swap scenarios was ever observed on the market

## Simulated swap rates



### Simulated swap rates



## Static simulation of credit spreads

1. Collect credit-spread indices on issuers with the same rating, currency, and sector
2. For each bond consider the corresponding index according to currency/sector and issuer rating according to the agencies
3. Apply the historical difference between the index spreads at two dates to the current credit spread

## Spread indices

<b>Sectors</b>	<b>Sub sectors</b>
all sectors	all sub sectors
financial	all financial insurance      finance      bank
industrial	all industrial consumer products      health care      media gaming and lodging      manufacturing      energy basic industries      technology      retail transportation      telecoms      property
public	all public agency      provincial      supranational sovereign      state guaranteed
utility	all utility electric      reg transp      gas pipelines



## Spread scenarios

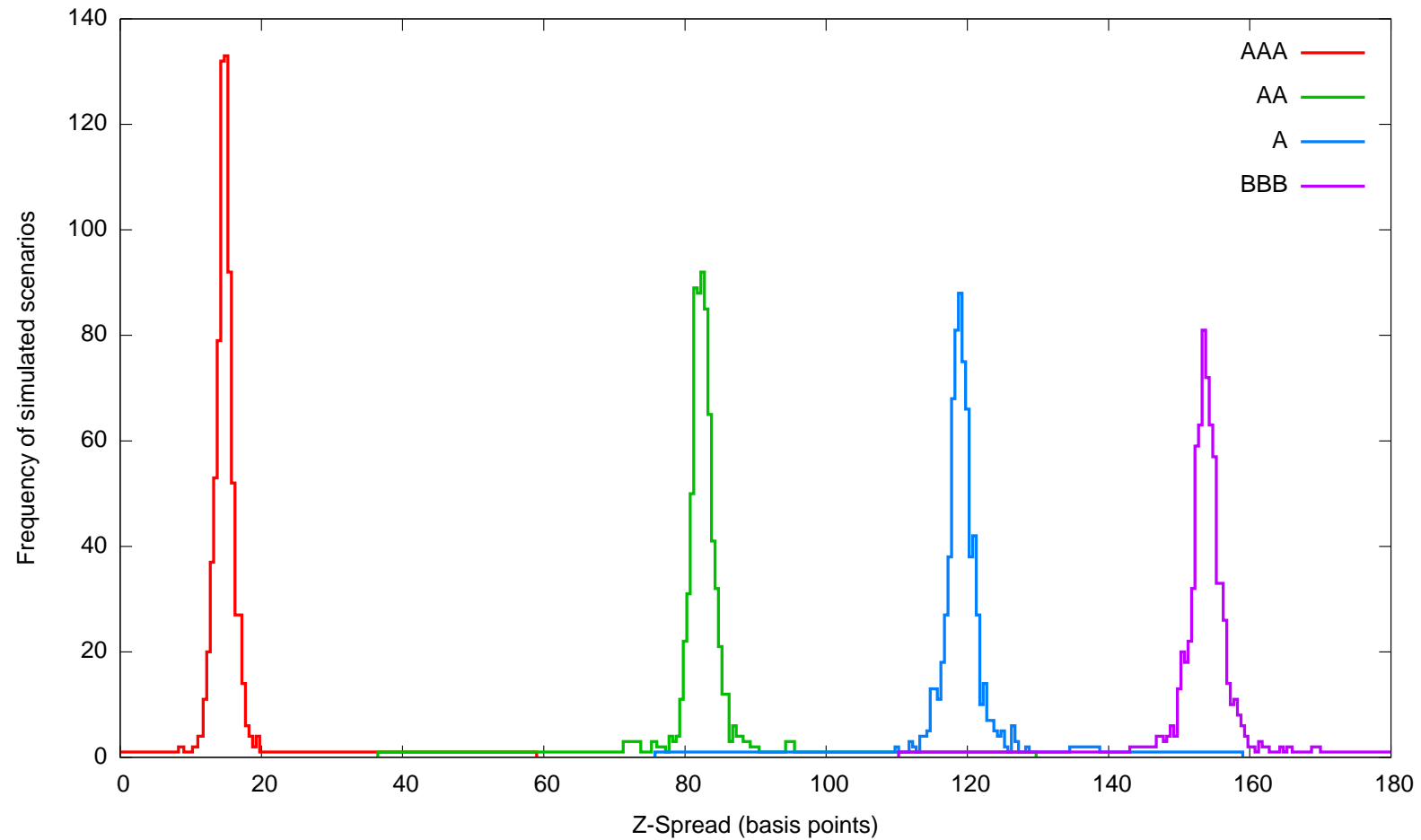
Given a maturity, a sector, a rating, and a currency from the historical spread indexes we can compute the daily variations

$$\Delta z_{AAA}^i = z_{AAA}^i - z_{AAA}^{i-1}; \quad \Delta z_{AA}^i = z_{AA}^i - z_{AA}^{i-1}; \quad \dots$$

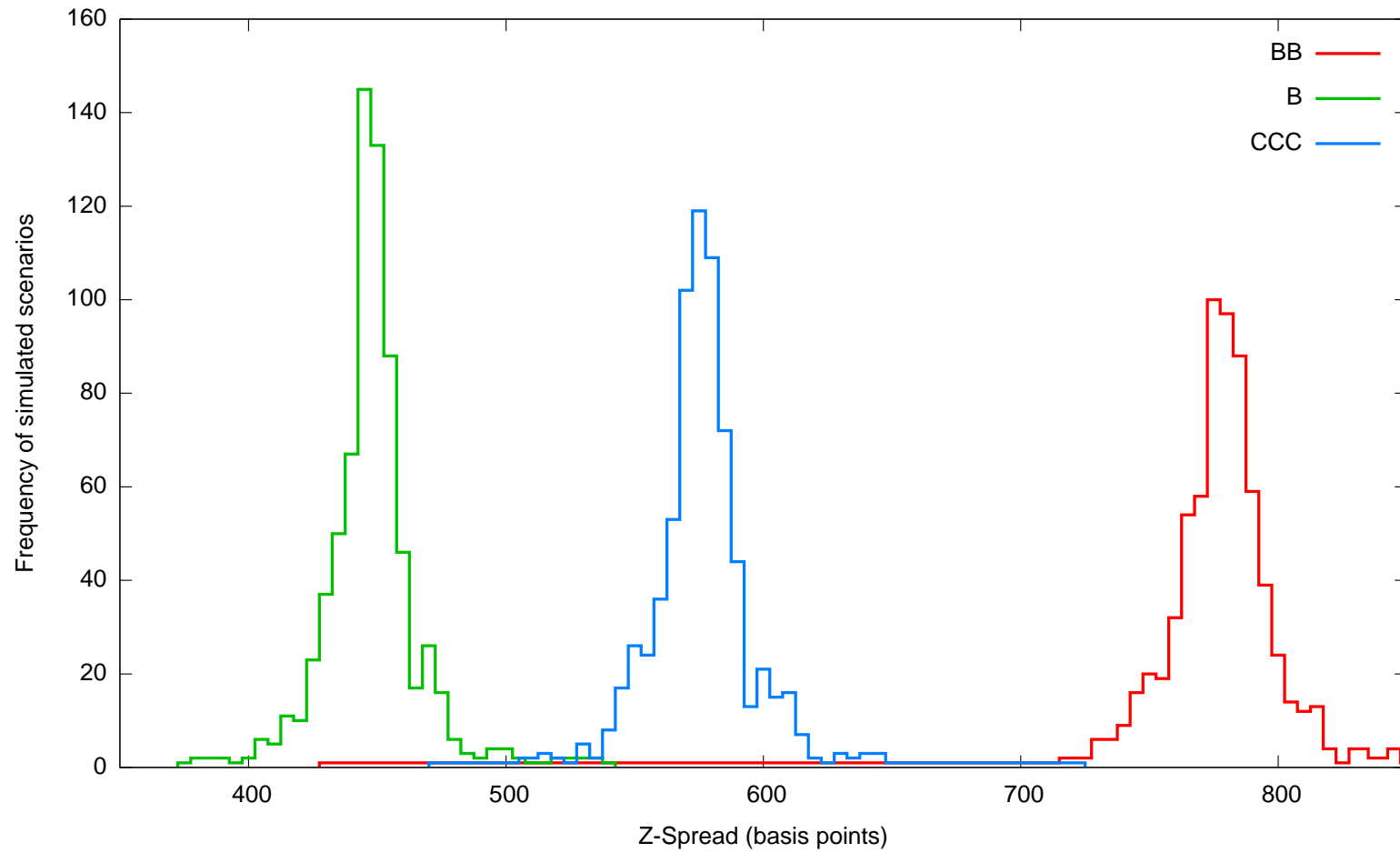
The spread scenarios are then obtained applying these variations to the current bond spread

$$z_{\text{sim}(AAA)}^i = z + \Delta z_{AAA}^i; \quad z_{\text{sim}(AA)}^i = z + \Delta z_{AA}^i; \quad \dots$$

## Simulated credit spreads: investment grades



Simulated credit spreads: high yield



## Static Methodology—The Good,

- As widely expected, the location of the reference spreads increases with the rating worsening
- As the rating decreases the probability distributions widens and the peaks become lower

## Static Methodology—The Bad, and ...

- Idiosyncratic risk is not captured as the pricing is performed using the average spread for the sector/rating/currency
- Two or more issuers in the same rating/sector/currency group will have similar spread scenarios and therefore their bond scenarios will be highly correlated
- The spread distribution reflects the rating (changing slowly) and not the market credit expectations (changing quickly)

## Dynamic simulations of credit spreads

- The observed bond spread is compared with the index spread for different ratings
- When the spread falls within two ratings we can determine a fractional rating
- Bond-spread simulations can be obtained mixing index-spread simulations for the bracketed ratings

## Fractional rating

Consider the credit spread for a bond, e.g.  $z = 48.9$ , and compare it with the index spreads for the same sector and currency for all ratings

$$z^0(\text{AAA}) = 15.0 < 48.9 < 82.7 = z^0(\text{AA})$$

Compute the spread fractions of each bracketing spread, in the example 50% and 50%, and define the fractional rating

$$50\% \text{ AAA} + 50\% \text{ AA}$$

For this bond we can define the spread scenarios as

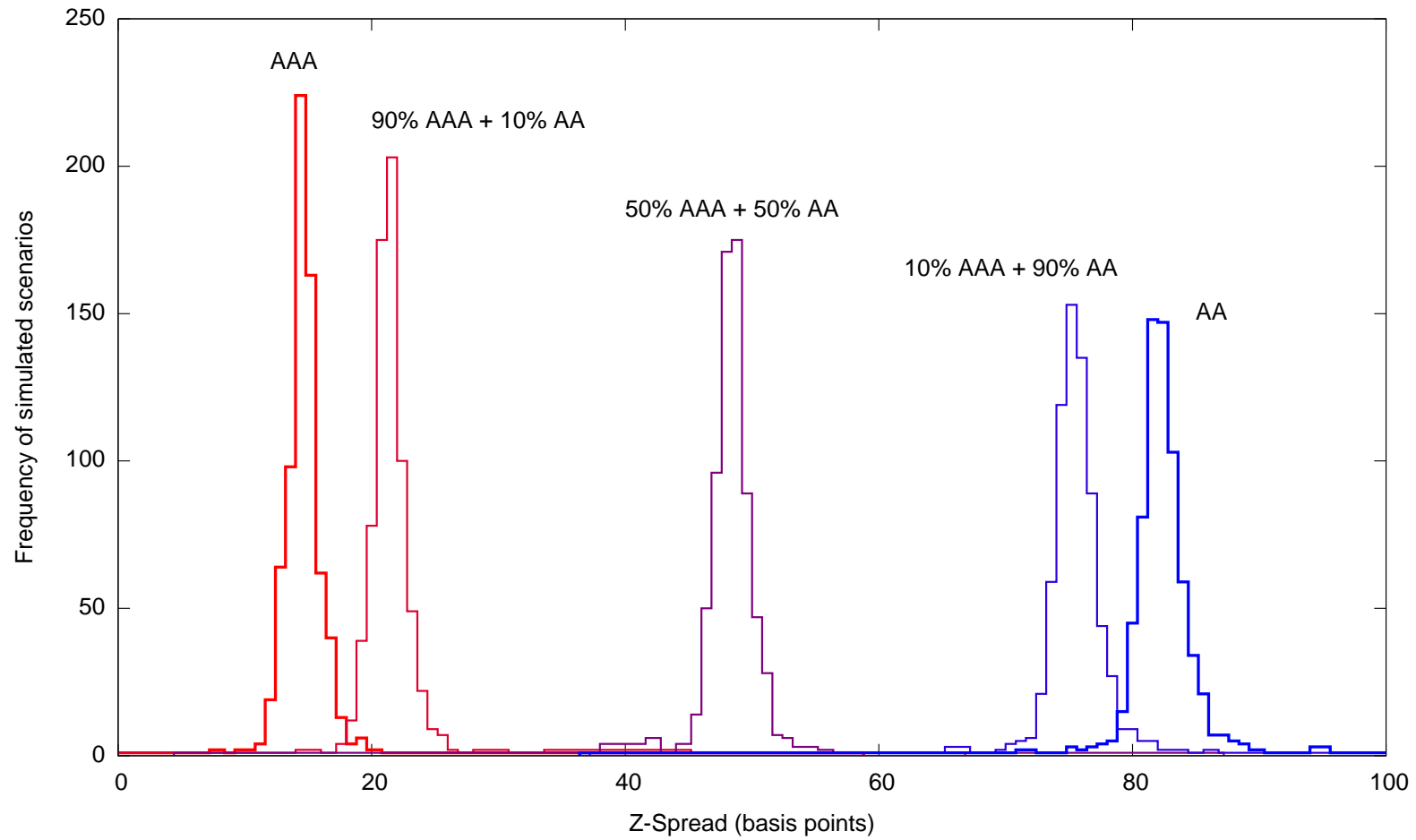
$$z^i = 50\% z_{\text{sim}(\text{AAA})}^i + 50\% z_{\text{sim}(\text{A})}^i$$

## Dynamic scenarios improvements

- The response to changes in credit deterioration, or improvement, are immediately reflected in a higher, or smaller, credit rating
- From day to day, as the fractional rating changes, the spread distribution changes in shape and position
- Spread scenarios evolve and become closer to the “*correct*” scenarios **without** the explicit intervention of rating agencies



Example: three different fractional ratings



## Historical simulations of bond prices

We show the StatPro version of the historical-simulation framework

- Instrument price depends deterministically from its risk factors
- In each simulation prices are *at equilibrium* and observed risk factor match simulated ones
- Risk factors are simulated starting from the current quote
- Bond prices computed using the simulated risk factors approximate the instrument distribution of future prices

## Pricing function for a coupon bond

Recall that for a bond with coupons  $C_1, \dots, C_n$  the current price is given by

$$P^0 = C_1 D_{\mathcal{S}^0}^{z^0}(T_1) + \dots + [C_n + R] D_{\mathcal{S}^0}^{z^0}(T_n)$$

Define the simulated prices as

$$P^i = C_1 D_{\mathcal{S}^i}^{z^i}(T_1) + \dots + [C_n + R] D_{\mathcal{S}^i}^{z^i}(T_n)$$

$D_{\mathcal{S}^i}^{z^i}$  is the discount curve obtained from the simulated swap rates  $\mathcal{S}_{\text{sim}}^i$  and spreads  $z_{\text{sim}}^i$

## Computation of risk statistics

Given the current bond price  $P^0$  and the simulated prices  $P^i$  we can consider the *distribution* of the simulated returns

$$\mathcal{D} = \left\{ \frac{P^i}{P^0} - 1 \right\}_{i=1, \dots, N}$$

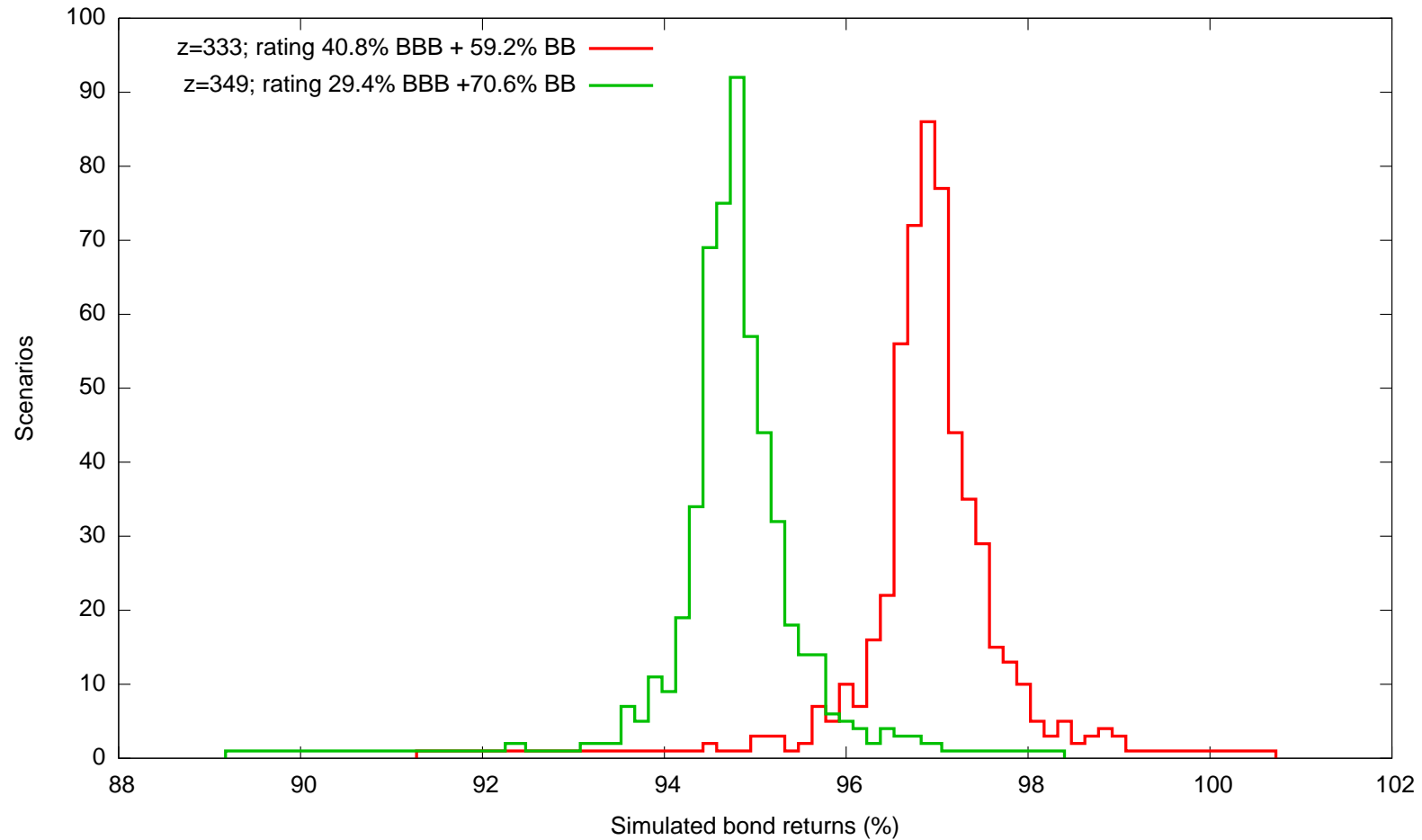
and compute the bond risk figures

$$\text{Value-at-Risk}(\mathcal{D}, 99\%) = 0.63\%$$

$$\text{Exp-Shortfall}(\mathcal{D}, 99\%) = 0.99\%$$

$$\text{Pot-Upside}(\mathcal{D}, 99\%) = 0.81\%$$

Simulated price distributions (“Fiat June 2017” and “Fiat Feb. 2021”)



## Risk statistics with the QuantLib addin

First create the statistic accumulator

```
stat = qlStatistics("stat", Values, Weights) -> "stat#0001"
```

Then compute the significant risk quantities

```
var95 = qlStatisticsValueAtRisk("stat", 0.95)  
es95 = qlStatisticsExpectedShortfall("stat", 0.95)  
pu95 = qlStatisticsPotentialUpside("stat", 0.95)
```

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## Generalization to a generic instrument

Given any (QuantLib) instrument with a pricing function  $f$

$$P^0 = f(t; X_1, \dots, X_K; r_1^0, \dots, r_n^0)$$

$X_1, \dots, X_K$  are static parameters

$r_1^0, \dots, r_n^0$  are the observed risk factors

The simulated scenarios at time  $T > t$  can be defined as

$$P^i = f(T; X_1, \dots, X_K; r_1^i, \dots, r_n^i)$$

$r_1^i, \dots, r_n^i$  are the simulated risk-factor scenarios for  $i = 1, \dots, N$

$N$  is the number of simulations



## Conclusions

- We looked into the details of scenario computation for swap curve and credit spread using dynamic model
- The dynamic model has been used in StatPro since early 2007 and has performed very well during the financial crisis
- We have shown how QuantLib can be used for price computations and for risk-management simulations of a fixed-rate coupon bond

- The method shown can be generalized to the computation of risk simulations for any instrument in QuantLib (and libraries based on QuantLib, e.g. StatPro's QuantLib<sup>2</sup>)