Risk decomposition for commodity futures spread

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Abstract

Financial markets recently rediscovered the importance of commodities as an asset class alternative to equities and bonds. More and more commodity-based derivatives are traded in the financial markets. We review the importance of convenience yield in commodity-derivative pricing; we describe the historical-simulation method in the commodity environment; finally, we describe the linear risk/upside decomposition of commodity futures spread and focus on numerical results.

Talk outline

1. Basic interest-rate results

2. Commodity futures and convenience yields

3. Price of some linear commodity derivatives

4. Risk simulations and linear risk decomposition

Part I: Basic interest-rate results

1. Day-count conventions and year fractions
2. The discount factor
3. Interest-rate curves
4. The forward rate and the FRA
5. Discount curves with QuantLib and QuantLibXL

Day-count conventions

We define the day-count convention used in order to compute year fractions

Given a date d_1 we use a day-count convention to associate to another date d_2, with $d_1 < d_2$, a year fraction $T(d_1, d_2)$.

For example the ISDA **Actual/365** defines the year fraction as

$$T_{\text{actual/365}} = \frac{n}{365} \tag{1}$$

with n the days between d_1 and d_2.

Assumptions on year fractions

We assume the year fraction T to have the following properties:

null at origin	$T(d_0, d_0) = 0$	d_0 the ref date
monotonicity	$T(d_0, d_1) < T(d_0, d_2)$	when $d_1 < d_2$
uniqueness	$T(d_0, d_1) = T(d_0, d_2)$	iff $d_1 = d_2$
additivity	$T(d_0, d_1) + T(d_1, d_2) = T(d_0, d_1)$	with $d_0 \leq d_1 \leq d_2$

The ISDA convention **Actual/365** satisfies these properties.

We create a one-to-one map between a date d and a year fraction $T = T(d_0, d)$ and use dates and year fractions interchangeably.

Discount factor

The discount factor is the present value of a future unit of currency

$$D(T) = \text{PV}(1 \text{ paid at } T) \tag{2}$$

with PV denoting the present-value operator

Using continuous compounding we define the discount factor

$$D(T) = e^{-z(T)\,T} \tag{3}$$

in terms of the interest rate $z(T)$, a.k.a. the **zero rate**

The map $z(T)$ is an **interest-rate curve**.

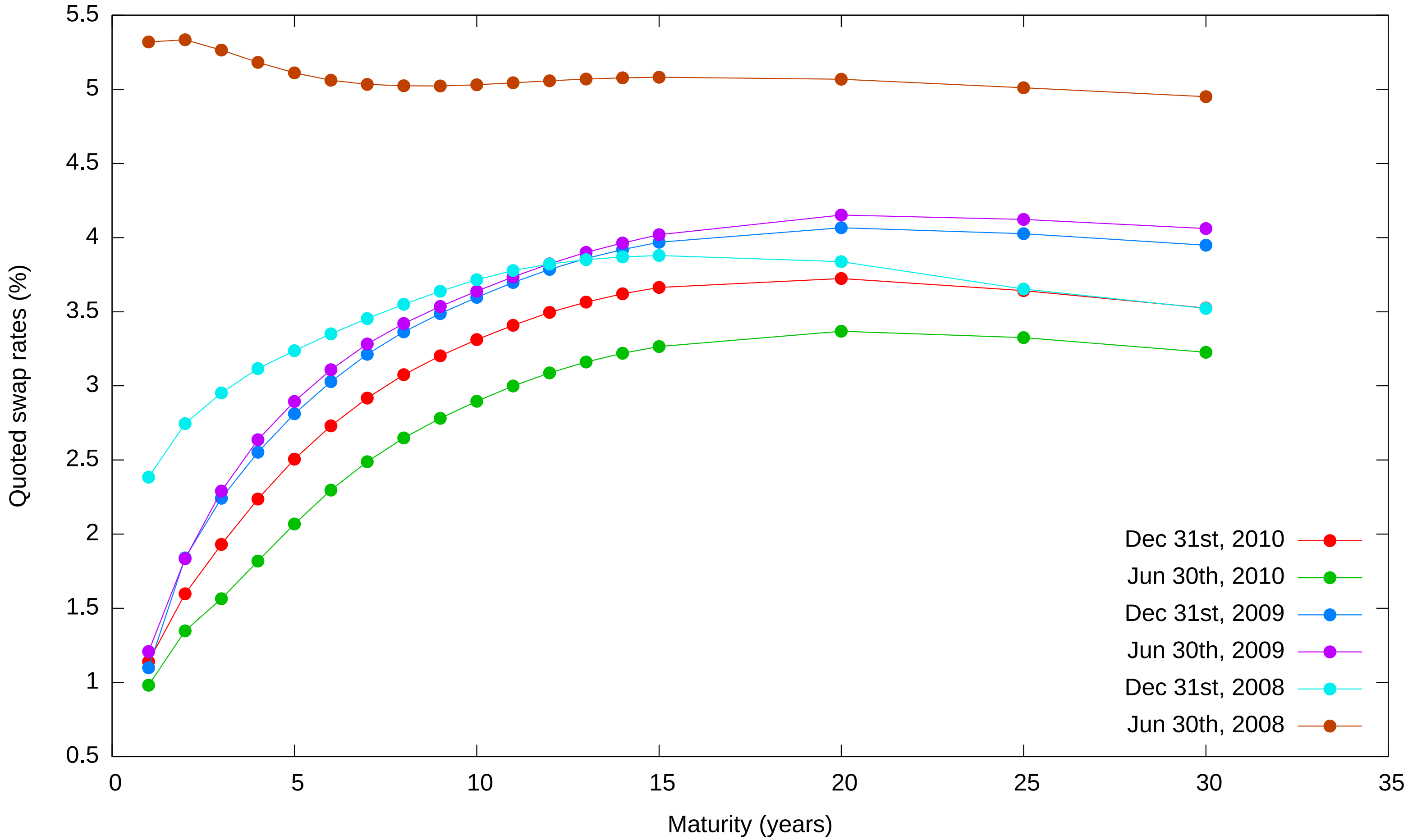
Interest rate curves

- Building block of financial engineering
- Used to compute the present value of cash flows paid at future known dates
- Should be calibrated from market quotes
- We assume the risk-free curve to be calibrated using first few deposit rates and swap rates
- In the old days used to be unique for each currency; now ...

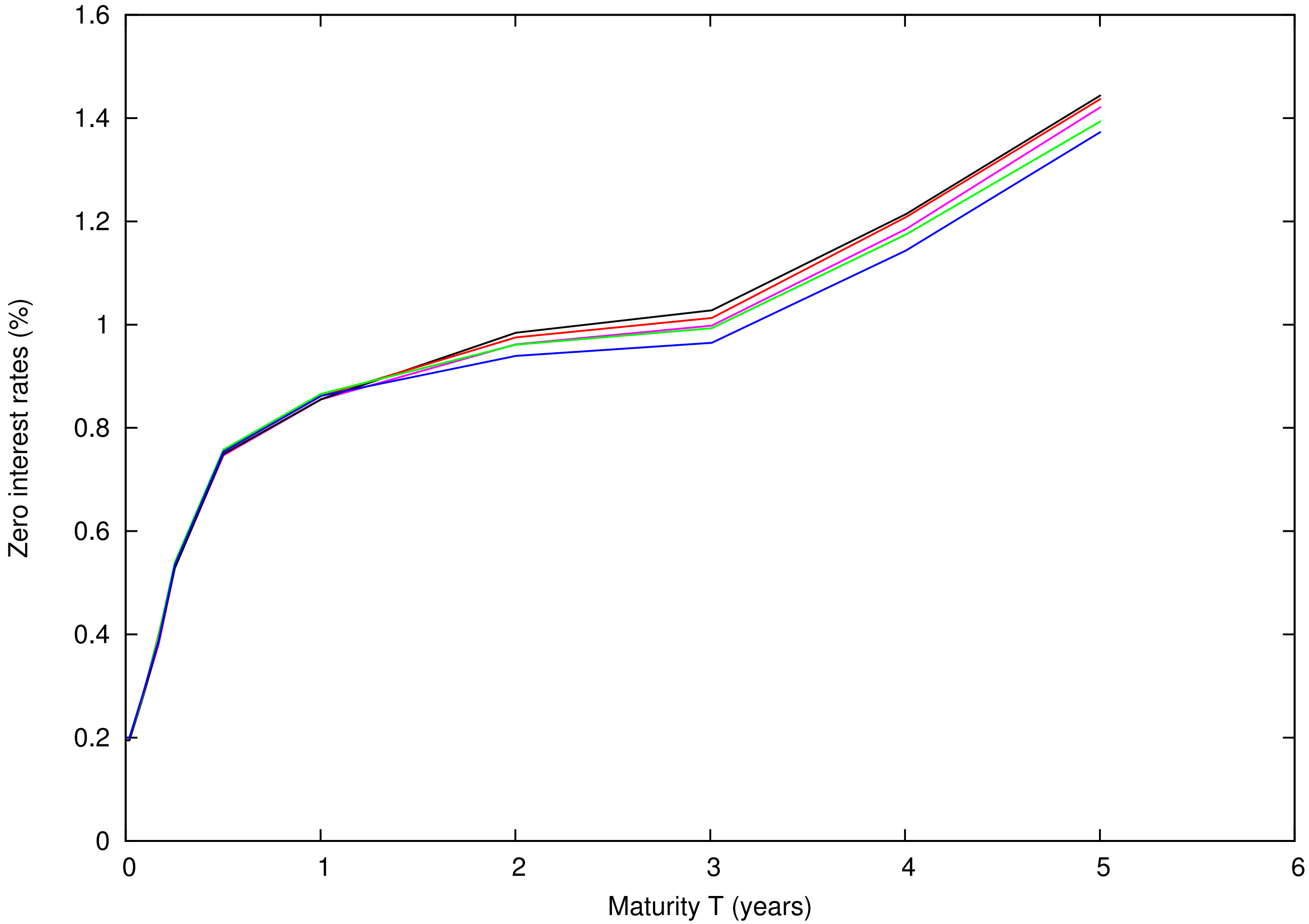
Bootstrap of a yield curve

- Choose the financial instruments to build the curve: quoted fair swap rates (or deposit rates, IR futures, ...)
- Choose a financial quantity to interpolate: discount rate (otherwise zero rate, forward rate, ...)
- Choose an interpolator, e.g. log-linear or something else, i.e. a parametric form for the curve
- Solve for the chosen parameters so that all instruments considered can be priced at the current market value

Historical swap rates (Eurirs)



Few zero interest-rate curves (USD, early November 2011)



Forward interest rates

Given the discount factor $D(T)$, there is a unique arbitrage-free future interest rate r_{12} between T_1 and T_2,

$$1 = \frac{D(T_2)}{D(T_1)} (1 + T_{12}\, r_{12}) \qquad \text{with } T_{12} = T_2 - T_1 \tag{4}$$

hence

$$r_{12} = \frac{1}{T_{12}} \left[\frac{D(T_1)}{D(T_2)} - 1 \right] \tag{5}$$

Forward rates can be used to compute the NPV of a FRAs.

Forward Rate Agreements

A FRA contract that locks in a rate r_{fra} between T_1 and T_2

The net present value (NPV) can be computed as

$$\text{NPV}_{\text{fra}} = D(T_1)\,c_1 + D(d_2)\,c_2 \tag{6}$$

with

$$c_1 = N \qquad \text{and} \qquad c_2 = -N\,(1 + r_{\text{fra}} T_{12}) \tag{7}$$

It can be shown that

$$\text{NPV}_{\text{fra}} = N\,D(T_2)\,T_{12}\,[r_{12} - r_{\text{fra}}] \tag{8}$$

QuantLib

- Available since November 2000 (Ametrano, Ballabio, Marchioro)
- Completely free (as in no charge, no constraints)
- On average about 15,000 downloads each year
- Used all-over the world (five continents)
- Implements the *standard* financial models
- Advanced and experimental implementations
- Great Excel interface (QuantLibXL)
- Object-oriented library

QuantLibXL

- Project started by Eric Ehlers (2004)
- Continually evolving
- Based on object-oriented programming
- Exposes QuantLib classes on an Excel interface
- Built on top of the QuantLibAddin: www.quantlibaddin.org
- In turn based on the Object Handler

Bootstrap of a swap curve in QuantLibXL

Create swap-rate helpers for the bootstrap

```
qlSwapRateHelper2("swap-10y",
      Rate, Tenor, Calendar, FixedLegFrequency,
      FixedLegConvention, FixedLegDayCounter, IborIndex,
      Spread, ForwardStart, DiscountingCurve) -> "swap-10y#0000"
```

Create and bootstrap the curve

```
qlPiecewiseYieldCurve("swap-YTS", NDays,
             Calendar, RateHelpers, DayCounter,
             Jumps, JumpDates, Accuracy, TraitsID,
             InterpolatorID) -> "swap-YTS#0000"
```

Discount factor using the QuantLibXL

In QuantLibXL the discount factor can be computed using

```
qlYieldTSDiscount("swap-YTS", DfDates,
                  AllowExtrapolation) -> 0.981536
```

and the forward rate as

```
qlYieldTSForwardRate("swap-YTS", D1, D2,
                     ResultDayCounter, Compounding,
                     Frequency, AllowExtrapolation) -> 0.023832
```

Part II: Commodities and convenience yields

1. Introduction to commodities

2. The forward-spot and the forward-forward relationship

3. Convenience yields

4. Example of commodity derivatives: The forward contract and the futures spread

Exposure to commodity prices

If you are managing an hedge fund and want some exposure on the price of live cattle what can you do?

A. Become a cowboy overnight and buy some live cattle

B. Enter in a derivative contract that gives you an exposure to the live-cattle price (e.g. a futures contract)

Commodity futures contracts

- Commodity futures are among the oldest financial instrument traded on any trading floor
- Futures on the cotton price, for example, have traded on the market for longer than a century
- Refer to specialized literature for more details
- In this talk we describe how to generate simulated spot prices and convenience-yield curves (to be used in the computation of risk figures)

Copper, Corn, and WTI Oil

We consider three samples commodities

- Copper (a.k.a Dr. Copper): non-ferrous industrial metal quoted, in $ per ton, on the London Metal Exchange (LME)

- Corn: an agriculture commodity quoted, in $-cents per bushel (27,216 kg), on the Chicago Board of Trade (CBoT)

- West-Texas-Intermediate Oil: an energy commodity quoted, in $ per barrel (158.987 liters), on the New York Mercantile Exchange (NYMEX)

Commodity futures contracts

A contract that allows the delivery of a commodity at a certain future date (however settled daily according to the close price).

E.g. on 2011-11-01 we observed the following market quotes

Commodity	**Maturity**	**Date**	**Price**
copper	2012G	2012-02-24	7993.00
copper	2012H	2012-03-27	7995.00
copper	2012J	2012-04-25	7995.50

Commodity forward contracts

A **commodity forward contract** is struck between two parties that agree to buy/sell a given commodity at a future date at a pre-determined price (the strike price).

Settlement could be physical or cash. In case of cash settlement the payoff P at the maturity date T (for the long side) is given by the difference between the value of the underlying commodity spot price $S(T)$ and a strike price K.

For W lots we have,

$$P = W \cdot \Big[S(T) - K\Big] \tag{9}$$

Arbitrage-free strategies

An arbitrage-free strategy is ...

A series of physical or financial transactions that starting with a portfolio with a zero value end up with a risk-less portfolio.

The no-arbitrage assumption states that the final portfolio value is zero (otherwise we could buy the cheaper part and sell the dearer one making a risk-free profit).

Spot-forward relationship (1/2)

Consider a forward contract on a certain storable commodity (no costs no benefits) asset and set up the following strategy:

Today borrow an amount S of currency, exactly enough to buy the spot asset (since T is a short maturity the borrowing can be made at reasonable, i.e. risk-free like, interest rates) and enter into a short forward contract to sell the asset at a future date T at a price f; then store the asset until the date T; when the date T comes, enforce the contract and sell the asset for a price f_T, finally, payback the loan with interests.

Spot-forward relationship (2/2)

The strategy can be summarized in the following table

Date	**Description**	**Cash flows**	**Asset exch.**
$t = 0$	borrow an amount S of cash	$+S$	
	purchase the asset at spot price	$-S$	+ asset
	enter into a short forward at T	0	
$0 < t < T$	store the asset		
$t = T$	use the asset to service the forward	f_T	- asset
	repay debt with interests	$-S[1 + R(T)\,T]$	

Because of no arbitrage we have

$$f_T = S\Big[1 + R(T)\,T\Big]\,, \tag{10}$$

Convenience yield

In the case of a commodity, for a dividend-paying stock, for a bond, and for some other asset types, in general there are costs and/or benefits associated in holding the asset,

$$f_T = S[1 + R(T)\,T] + \mathrm{Cos}(T) - \mathrm{Ben}(T) \tag{11}$$

Assuming costs and benefits to be proportional to the asset price, for continuous compounding we can write

$$f_T = \frac{S}{D(T)} e^{-y\,T} \tag{12}$$

in general the **convenience yield** y depends on the maturity T.

Forward-forward relationship

Strategy: at time $t = 0$ enter into a long forward contract to buy the asset at T_1 for f_1 units of currency, at the same time enter into a short contract to sell the asset at date T_2 for a price f_2; at date $t = T_1$ use the long forward contract to buy the asset for a price of f_1 financing the purchase by borrowing the money from the market; at a later date T_2 sell the commodity for f_2

From arbitrage-free assumption we have

$$f_2 = f_1 \, \frac{D(T_1)}{D(T_2)} \, e^{-y_{12}\,(T_2 - T_1)} \qquad (13)$$

where y_{12} is the **forward convenience yield** between T_1 and T_2.

General spot-forward relationship

The forward-forward relationship between T_2 and T_3 is

$$f_3 = f_2 \, \frac{D(T_2)}{D(T_3)} \, e^{-y_{23}\,(T_3 - T_2)} \tag{14}$$

Chain linking the spot-forward formula and multiple forward-forward relationships we can write

$$f_j = \frac{S}{D(T_j)} e^{-y_j\,T_j} \tag{15}$$

where

$$y_j\,T_j = y_1\,T_1 + y_{12}\,(T_2 - T_1) + y_{23}\,(T_3 - T_2) + \ldots + y_{ij}\,(T_j - T_i)$$

Forward-futures relationship

The main difference between a futures contract and a forward contract is that the former must be settled daily while the latter is settled at the contract maturity. It can be shown that we have,

$$F^T = f^T + D(T)\,\sigma_S\,\sigma_r\,\rho_{r-S}\,, \tag{16}$$

- σ_s is the asset-price volatility
- σ_r is the interest-rate volatility
- ρ_{S-r} is the correlation between the asset price and the money-market account.

Convenience yields from futures quotes

We assume $|\sigma_S \, \sigma_r \, \rho_{r-S}| \ll 1$ so that

$$F^T \simeq f^T \tag{17}$$

Consider n futures contracts with maturities $T_1, T_2, \ldots T_n$, having, respectively, quotes F_1 $F_1, F_2, \ldots F_n$ at some reference date.

From equation (15) and (17) we have

$$F_i = \frac{S}{D(T_i)} e^{-y_i T_i} \,, \qquad \text{for } i = 1, \ldots, n \,, \tag{18}$$

compute the convenience yields y_i's as

$$y_i = \frac{1}{T_i} \log \left(\frac{S}{D(T_i) F_i} \right) \tag{19}$$

Convenience yields without spot quotes (1/2)

Very often spot and futures quotes are not aligned. Using $F^{T_i} = f^{T_i}$ and the forward-forward relationship we have

$$F_2 = \frac{D(T_1)}{D(T_2)} F_1 \, e^{-y_{12}\,(T_2 - T_1)}\,, \tag{20}$$

with T_1 and T_2 the next and the second-next maturities.

Hence,

$$y_{12} = \frac{1}{T_2 - T_1} \log\left(\frac{D(T_1)\,F_1}{D(T_2)\,F_2}\right) \tag{21}$$

is the (continuously-compounded) forward convenience yield

Convenience yields without spot quotes (2/2)

Assume $y_1 = y_{12}$ so that

$$S^f = D(T_1)\, F_1\, e^{y_{12} T_1}\,. \tag{22}$$

Using also (19) we can compute the y_i's without the need of S:

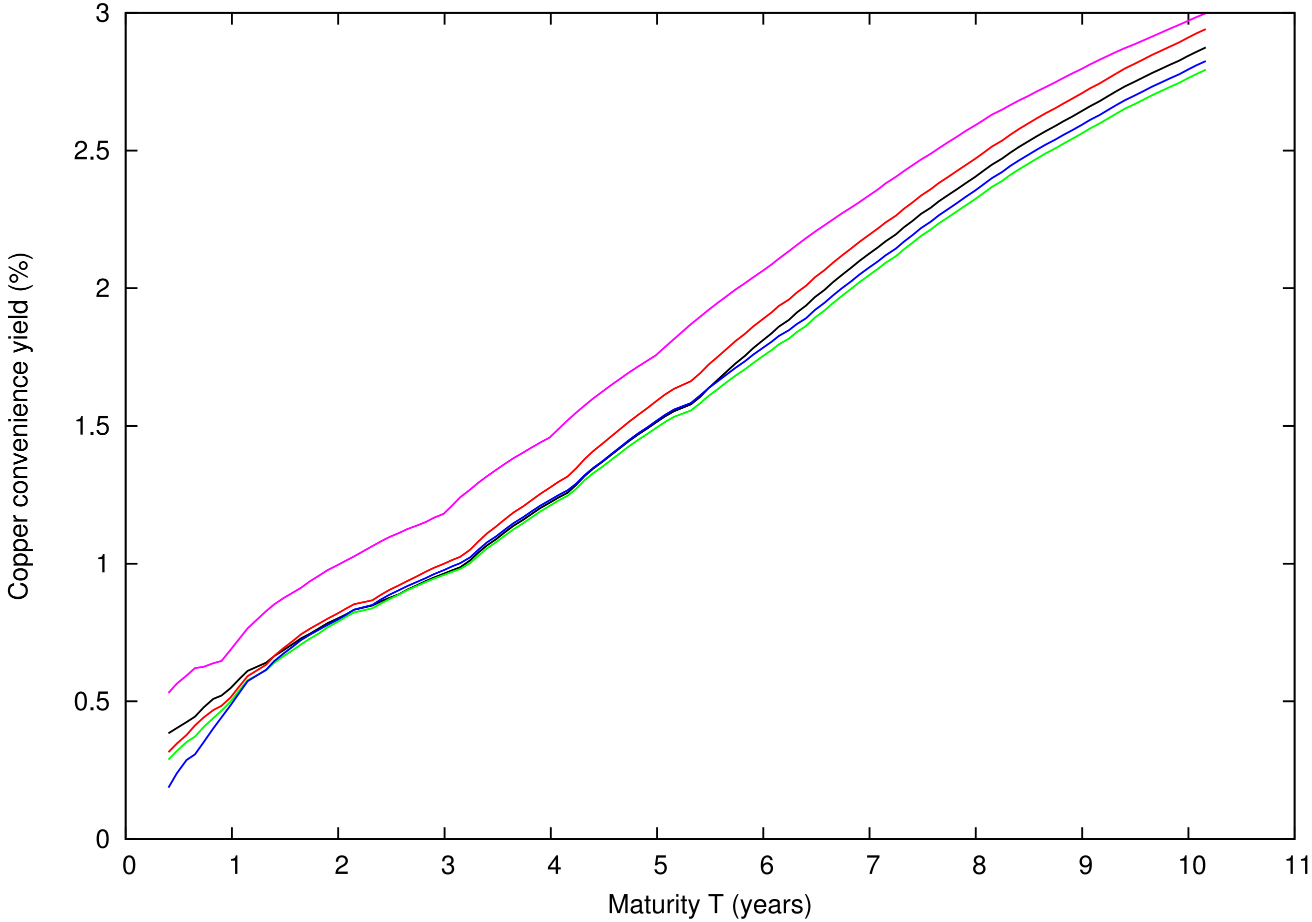
$$y_i = \frac{1}{T_i} \log\left(\frac{S^f}{D(T_i) F_i}\right) = \frac{1}{T_i} \log\left(\frac{D(T_1)\, F_1}{D(T_i)\, F_i}\right) + \frac{T_1}{T_i} y_{12}\,. \tag{23}$$

Given the y_i's we interpolate them to obtain a smooth **convenience yield curve** $y(T)$.

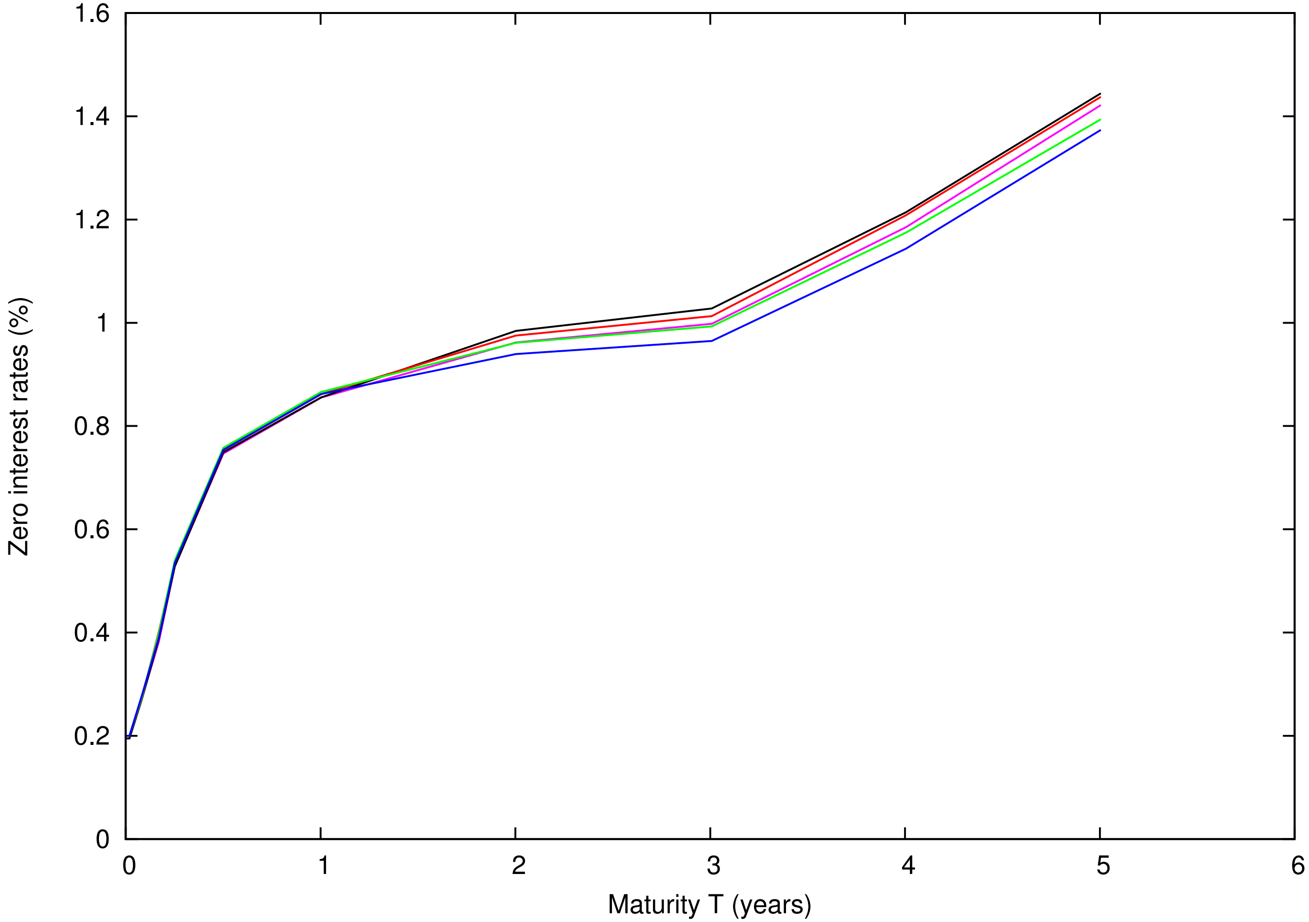
Implied commodity spot prices

Observation date	**Copper**	**Corn**	**WTI Oil**
2011-11-01	7993.52	655.57	92.44
2011-11-02	7730.52	643.75	92.62
2011-11-03	7883.80	652.11	94.20
2011-11-04	7914.73	652.11	94.46
2011-11-07	7822.33	650.43	95.60
2011-11-08	7796.89	656.81	96.99
2011-11-09	7619.39	651.95	95.96

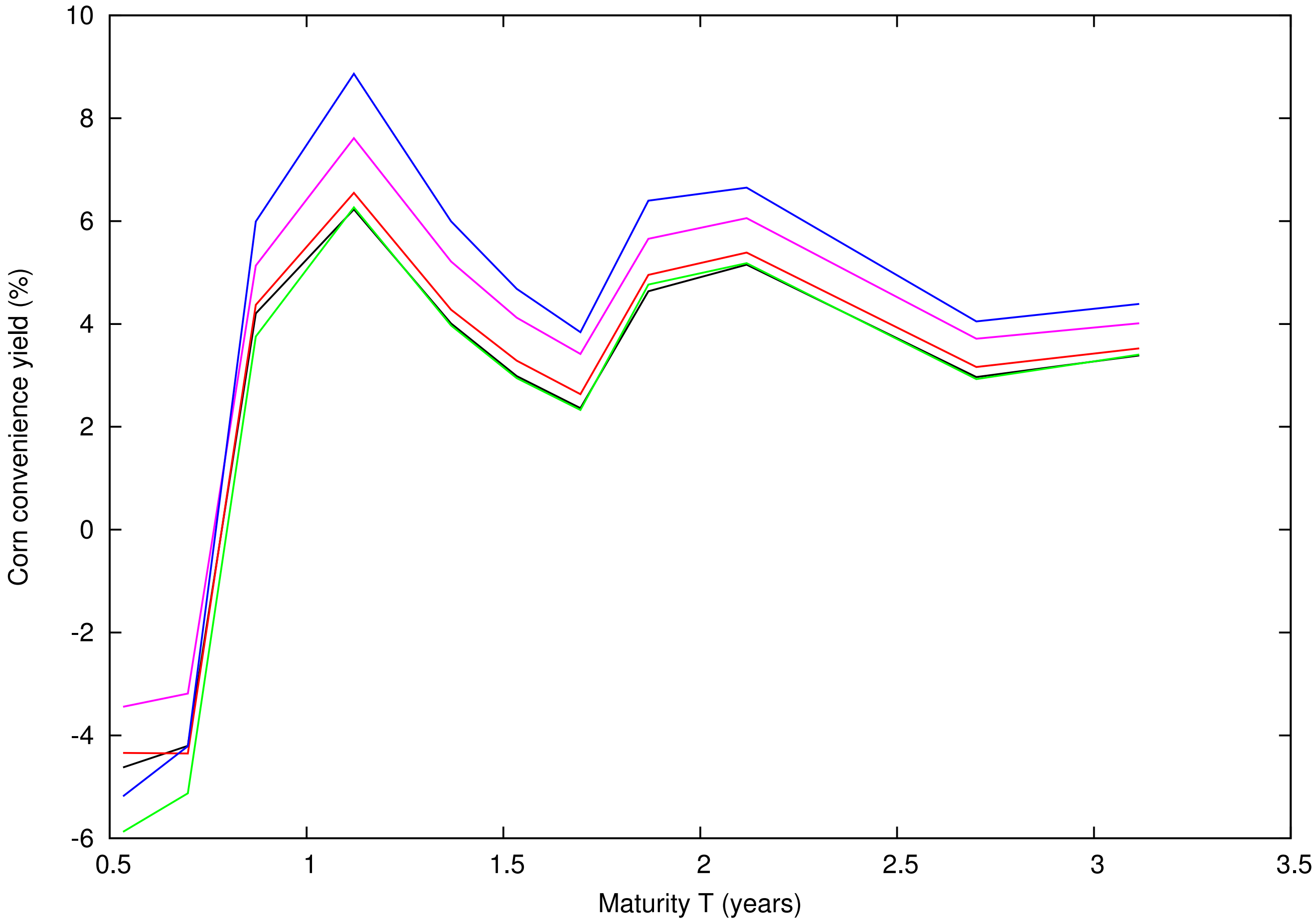
Few convenience-yield curves for Copper



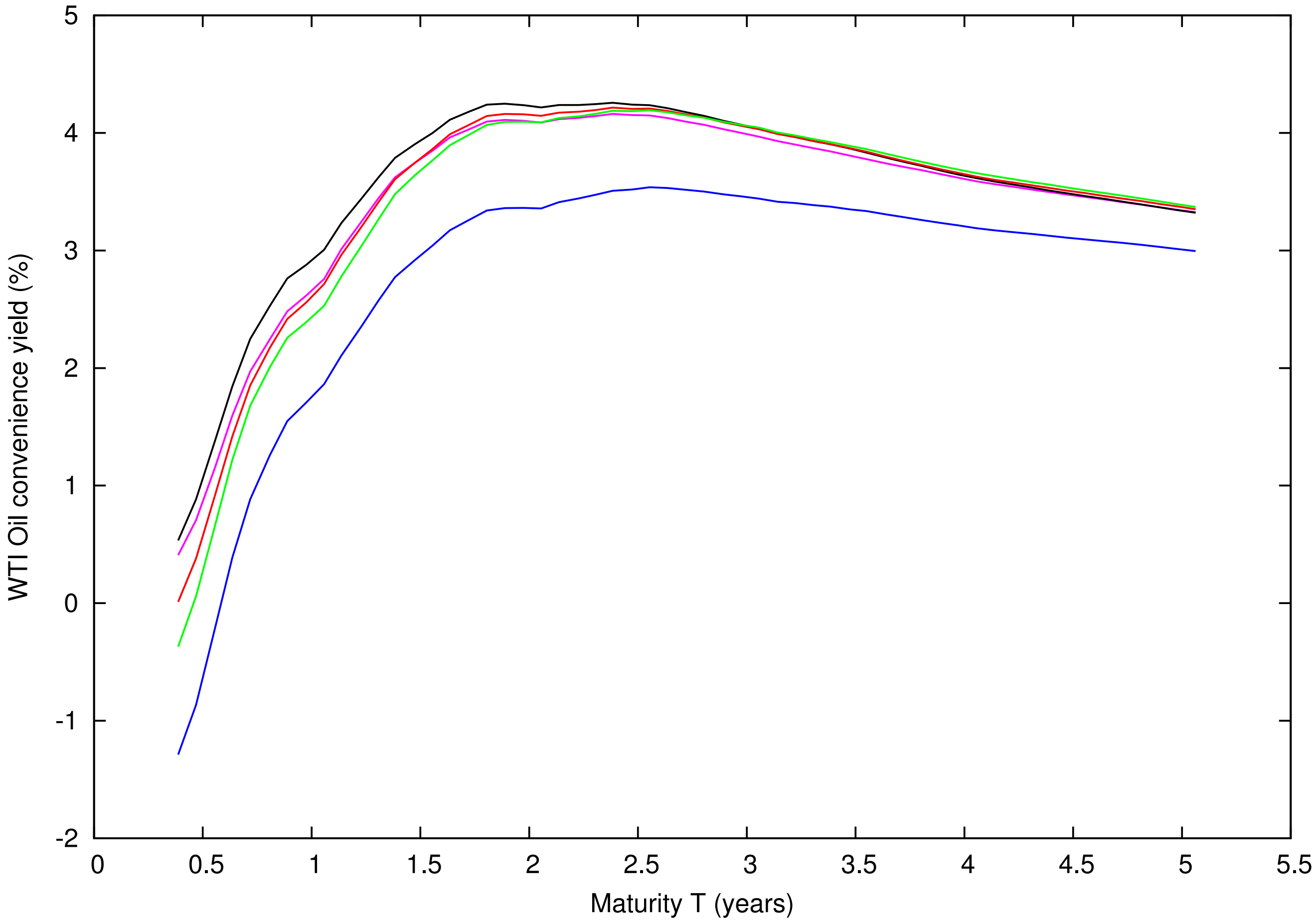
Recall the zero interest-rate curves



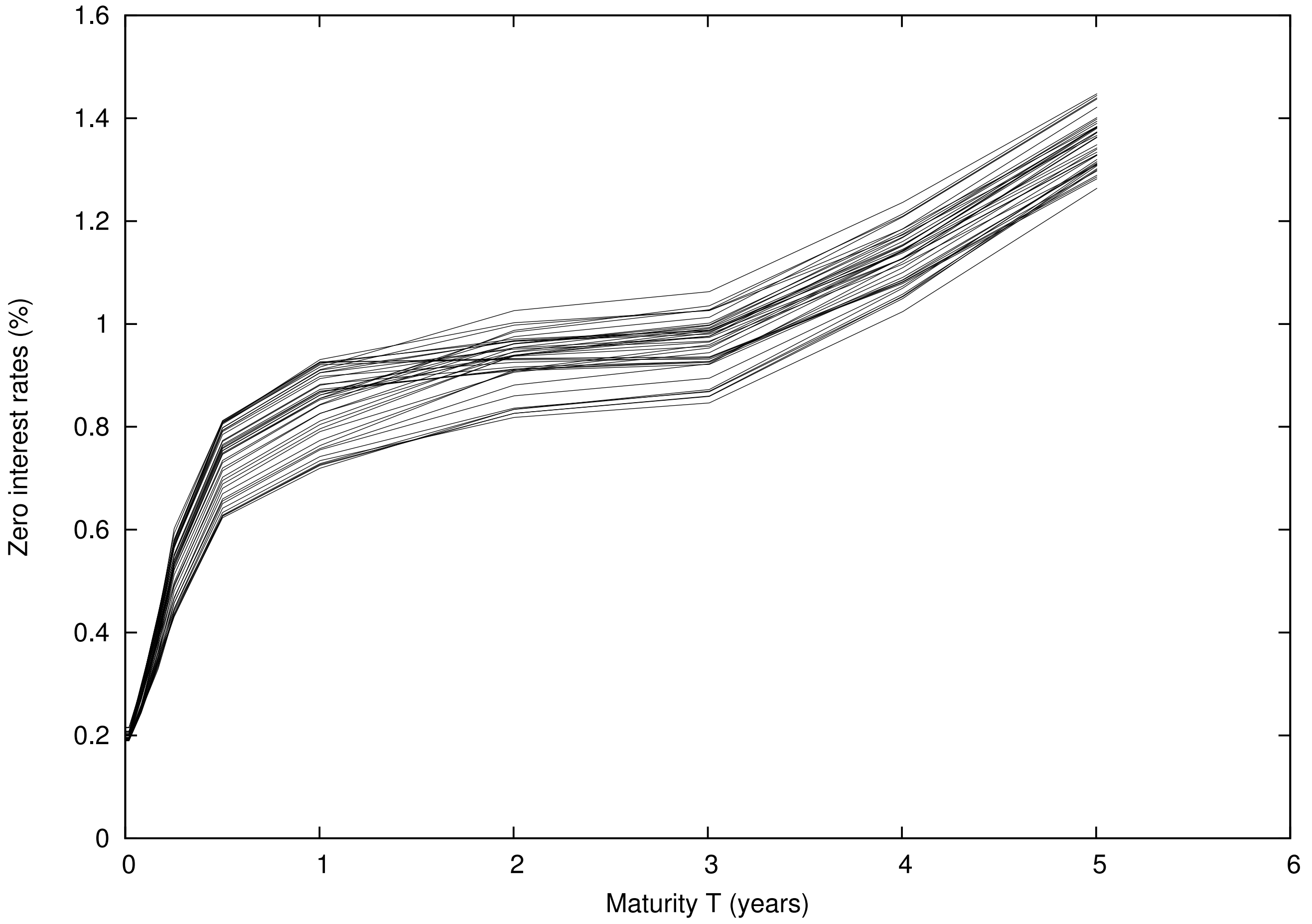
Few convenience-yield curves for Corn



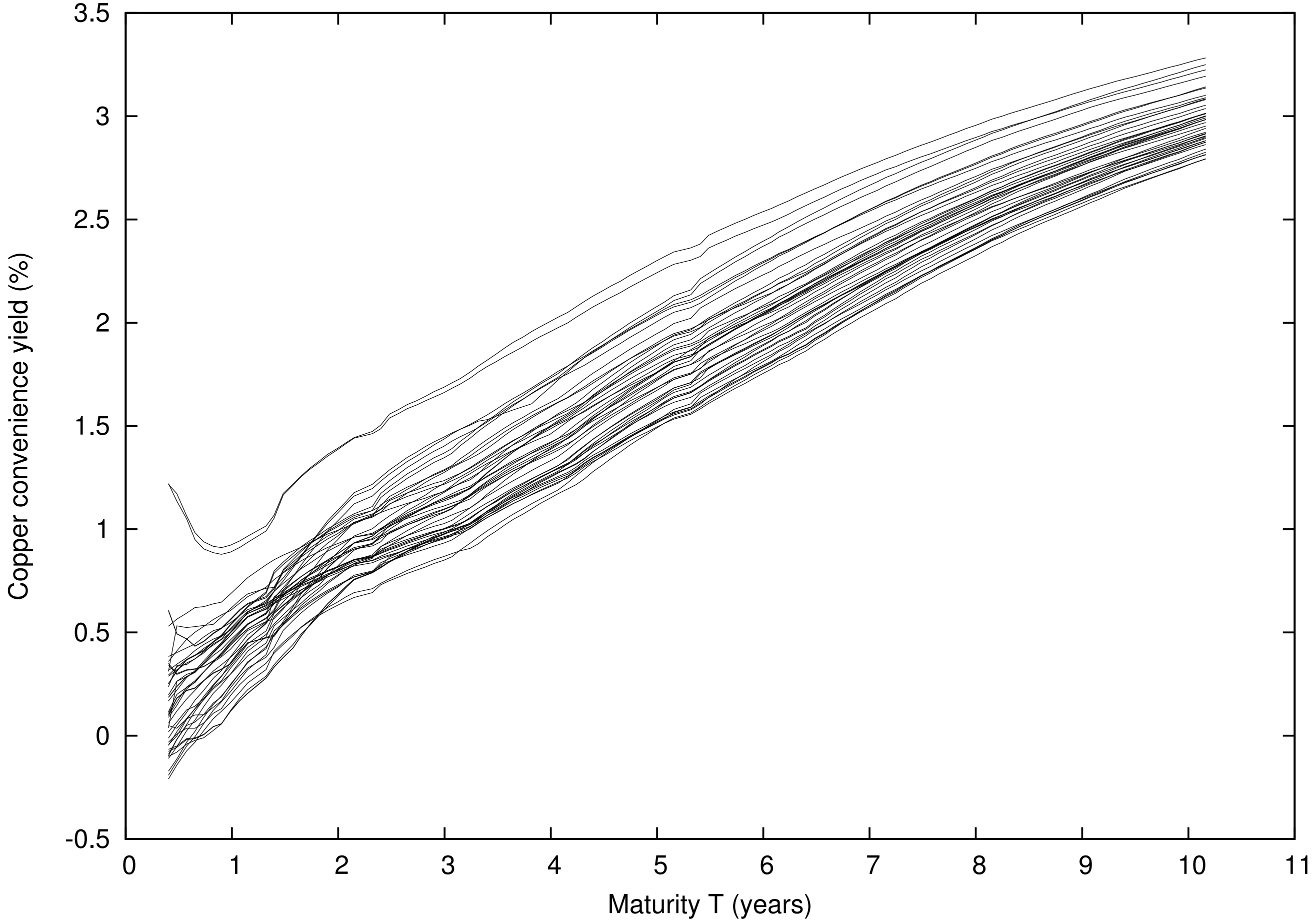
Few convenience-yield curves for WTI Oil



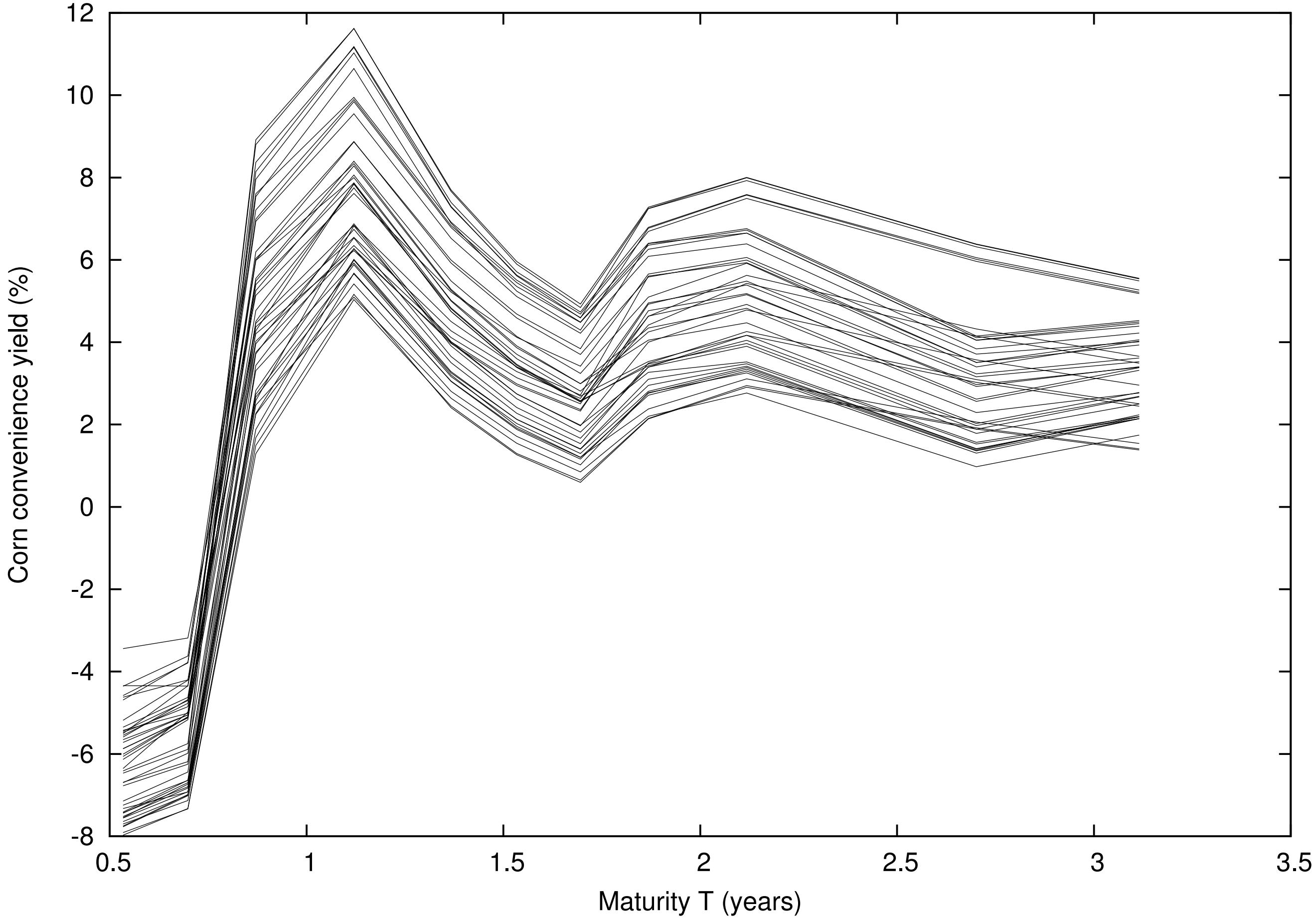
Many zero interest-rate curves



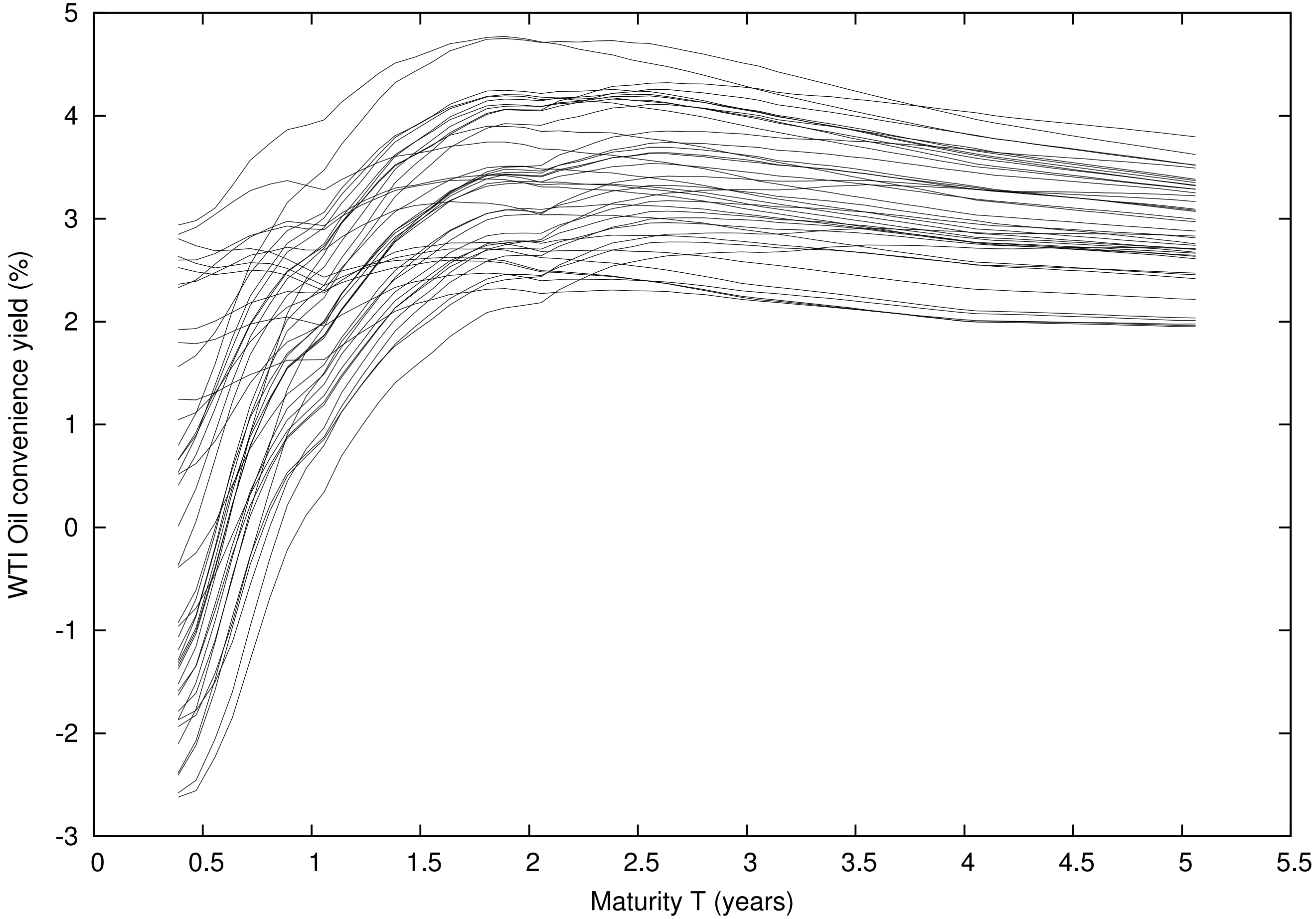
Many convenience-yield curves for Copper



Many convenience-yield curves for Corn



Many convenience-yield curves for WTI Oil



Part III: Price of linear commodity derivatives

1. The of a commodity forward contract

2. The price of a commodity futures spread

3. Numerical examples

NPV of a commodity forward contract

Given the convenience yield curve $y(T)$ define the (continuously-compounded) **convenience discount factor** $D^c(T)$ as

$$D^c(T) = e^{-y(T)\,T}, \tag{24}$$

so that the **commodity forward price** can be written as

$$f(T) = S\frac{D^c(T)}{D^r(T)}. \tag{25}$$

The commodity forward contract NPV is then given by

$$\text{NPV} = W \cdot D^s(T) \cdot \Big[f(T) - K\Big]. \tag{26}$$

At inception NPV=0 implies $K = f(T)$.

Short-maturity and long-maturity commodity futures

Assuming equivalence between forward prices and futures price

$$F(T_1) = S\,\frac{D^c(T_1)}{D^r(T_1)} = S\,e^{(z-y)T_1} \simeq S\left[1 + (z-y)T_1\right].$$

For short maturities $T_1 \sim 0.1$, assuming $|z-y| \sim 5\%$ we have

$$F(T_1) \sim S\left[1 \pm 0.5\%\right],$$

we are mainly exposed to the spot commodity price.

For long maturities T_2

$$F(T_2) = F(T_1) - \left[F(T_1) - F(T_2)\right] \simeq S - \text{futures spread}\,,$$

we are exposed to commodity prices and to a (short) futures spread

Commodity futures spread (1/2)

A futures-spread contract is equivalent to a long futures position at T_1 and short one at T_2

$$\begin{aligned} P &= F_{T_1} - F_{T_2} = S \cdot \left[\frac{D^c(T_1)}{D^r(T_1)} - \frac{D^c(T_2)}{D^r(T_2)} \right] \\ &= S \frac{D^c(T_1)}{D^r(T_1)} \cdot \left[1 - \frac{D^c(T_2)}{D^c(T_1)} \cdot \frac{D^r(T_1)}{D^r(T_2)} \right] = F_{T_1} \cdot \left[1 - \frac{1 + r_{12}\, T_{12}}{1 + y_{12}\, T_{12}} \right] \end{aligned}$$

where $T_{12} = T_2 - T_1$ and the (simply-compounded) *forward convenience yield* defined as

$$y_{12} = \frac{1}{T_{12}} \left[\frac{D^c(T_1)}{D^c(T_2)} - 1 \right] . \qquad (27)$$

Commodity futures spread (2/2)

Usually $T_{12} < 1$ so that $|T_{12}\, r_{12}|$ and $|T_{12}\, y_{12}|$ are both small, hence

$$
\begin{aligned}
P &= F_{T_1} - F_{T_2} = F_{T_1} \cdot \left[1 - \frac{1 + r_{12}\, T_{12}}{1 + y_{12}\, T_{12}}\right] \\
&\simeq F_{T_1} \cdot \left[1 - (1 + r_{12}\, T_{12})\,(1 - y_{12}\, T_{12})\,\right] \\
&\simeq F_{T_1} \cdot T_{12} \cdot \left[y_{12} - r_{12}\right]
\end{aligned}
$$

The price of a futures spread is proportional to F_{T_1} and the difference between the convenience yield and the interest rate (both forward).

Futures-spread contracts

We consider three futures-spread contracts

Contract	**Futures side**	**Maturity**	**Date**
Copper	long	2012N	2012-07-26
futures spread	short	2013N	2013-07-26
Corn	long	2012N	2012-07-13
futures spread	short	2013N	2013-07-12
WTI-Oil	long	2012N	2012-07-20
futures spread	short	2013N	2013-07-20

Copper contract

	T_1 **value**	T_2 **value**	$T_1 \to T_2$
copper	8373.00	8350.00	
T	0.51	1.51	1.00
y	0.50%	0.82%	0.99%
z	0.81%	0.74%	0.71%

$$P = F(T_1) \cdot \left[1 - \frac{1 + r_{12} T_{12}}{1 + y_{12} T_{12}}\right] = 23.00\,. \tag{28}$$

$$\frac{P}{F(T_1)} = 0.27\%\,. \tag{29}$$

Corn contract

	T_1 **value**	T_2 **value**	$T_1 \to T_2$
corn	629.75	581.00	
T	0.47	1.47	1.00
y	-4.02%	4.71%	9.23%
z	0.78%	0.76%	0.75%

$$P = F_{T_1} \cdot \left[1 - \frac{1 + r_{12}T_{12}}{1 + y_{12}T_{12}}\right] = 48.75\,. \tag{30}$$

$$\frac{P}{F(T_1)} = 7.74\%\,. \tag{31}$$

WTI-Oil contract

	T_1 **value**	T_2 **value**	$T_1 \to T_2$
wti-oil	100.64	98.66	
T	0.41	1.41	1.00
y	-2.53%	1.26%	2.84%
z	0.75%	0.79%	0.82%

$$P = F_{T_1} \cdot \left[1 - \frac{1 + r_{12}\, T_{12}}{1 + y_{12}\, T_{12}}\right] = 1.98\,. \tag{32}$$

$$\frac{P}{F(T_1)} = 1.97\%\,. \tag{33}$$

Part IV: Risk simulations

1. Simulations of commodity spot prices

2. Simulations of interest-rate curves and convenience-yield curves

3. Weekly pricing-function simulations

4. Risk/upside results and risk/upside decomposition

Risk-management simulations

Given a financial instrument we generate some realistic scenarios of its value at a future date (e.g. next week)

- Consider the financial variables affecting the price: the risk factors
- Create simulation scenarios for each risk factor and compute the pricing function assuming the realization of each scenario
- The array of computed values can be used to compute risk-management numbers such as **value at risk**

Daily historical simulations of commodity prices

Given the price fixings (or futures implied) S_k^{hist}'s for a number of historical dates t_k's, compute the performance ratios between consecutive days

$$\Pi^k = \frac{S_k^{\text{hist}}}{S_{k-1}^{\text{hist}}} \tag{34}$$

multiply the reference price by these ratios to obtain the price scenarios

$$S_k^{\text{scen}} = S_{\text{ref}} \cdot \Pi^k \tag{35}$$

Daily historical simulations of WTI-Oil prices

Historical data from 2011-11-01 to 2012-01-23

Obs. date	**WTI Oil**	**Ratio**	**Scenario**
2011-11-01	92.44		
2011-11-02	92.62	1.0019	99.49
2011-11-03	94.20	1.0171	100.99
2011-11-04	94.46	1.0028	99.57
2011-11-07	95.60	1.0121	100.50
2011-11-08	96.99	1.0145	100.74
2011-11-09	95.96	0.9894	98.25
...	...	...	...
2012-01-20	97.98	0.9774	97.05
2012-01-23	**99.30**	1.0135	100.64

Historical simulations of swap curves

We do not know what the swap rates will be tomorrow or next week, however, based on the recent past, we can simulate a number of possible future scenarios for tomorrow's swap rates.

Assumptions:

1. Tomorrow's swap rates will be similar to today's rates
2. The (absolute) variations of swap rates observed historically from day to day are applied to current swap rates to create the scenarios
3. In each scenario assume the swap rates as simulated and bootstrap the interest-rate curve to obtain a discount curve

(Proceed similarly for deposit rates or other quoted instruments.)

Daily historical scenarios for swap rates

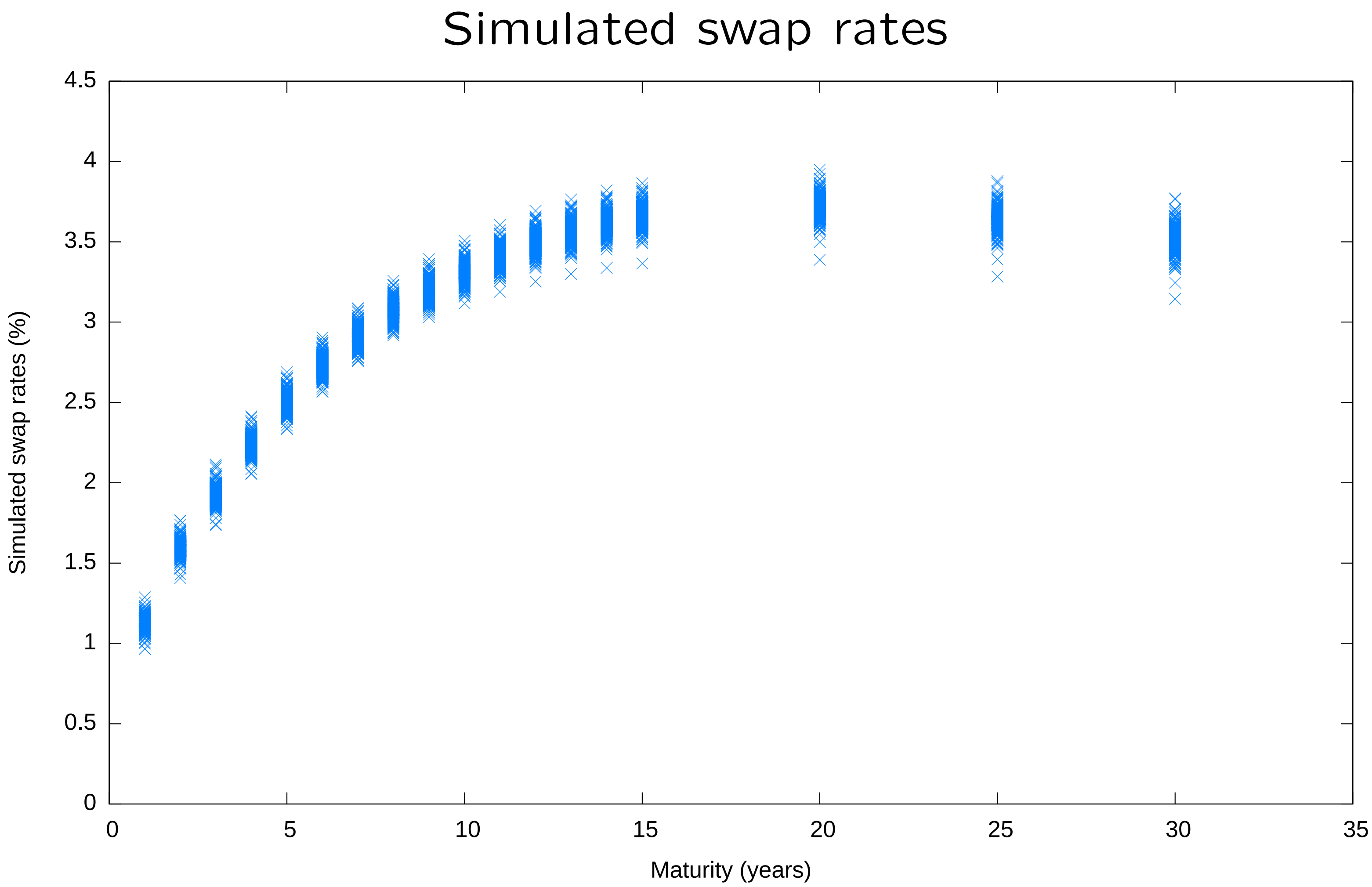
For each historical date t_k consider the increment in the swap rates with respect to the previous day

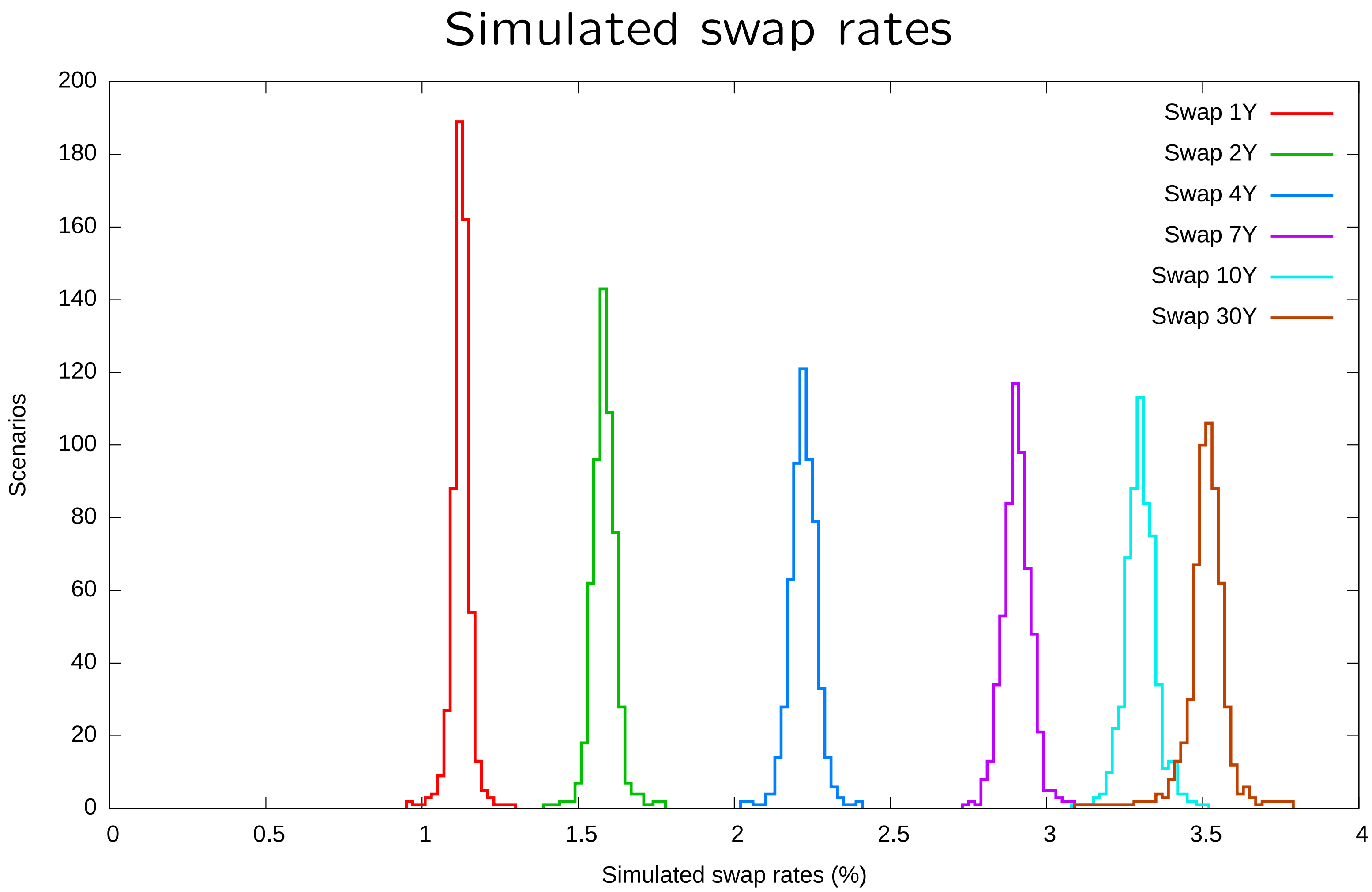
$$\Delta S_k^{1Y} = S_k^{1Y} - S_{k-1}^{1Y} \qquad \ldots \qquad \Delta S_k^{30Y} = S_k^{30Y} - S_{k-1}^{30Y}$$

Apply these increments to the reference rates

$$S_k^{\text{scen}(1Y)} = S_{\text{ref}}^{1Y} + \Delta S_k^{1Y} \qquad \ldots \qquad S_k^{\text{scen}(30Y)} = S_{\text{ref}}^{30Y} + \Delta S_k^{30Y}$$

Note: none of the swap scenarios was ever observed on the market


Simulated swap rates
Simulated swap rates (%)
Maturity (years)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
5
10
15
20
25
30
35


Simulated swap rates
Scenarios
Simulated swap rates (%)
Swap 1Y
Swap 2Y
Swap 4Y
Swap 7Y
Swap 10Y
Swap 30Y
0
20
40
60
80
100
120
140
160
180
200
0
0.5
1
1.5
2
2.5
3
3.5
4

Alternative: simulation of zero rates

One may decide to actually perform simulations of zero rates instead of swap rates

The choice between the two method should be made by the final user, however, usually

- Fixed-income traders prefer swap simulations
- Bond traders and asset managers have a preference for zero-rate simulations

Daily scenarios for convenience-yield curves

Convenience yield curves at different dates are not directly comparable: we need a change of the time variable from the *maturity date* to the *time to maturity*.

Consider a past date t_k and the observed convenience-yield curve $y^k(\tau)$ as a function of the time to maturity $\tau = T - t_k$. Compute the **daily convenience-yield increment** as

$$\Delta\gamma^k(\tau) = y^k(\tau) - y^{k-1}(\tau) \qquad \text{for all } t^k$$

define the **daily historical convenience-yield scenario** as

$$y_k^{\text{scen}}(\tau) = y^{\text{ref}}(\tau) + \Delta\gamma^k(\tau)$$

Weekly scenarios

Given the daily scenario we can derive (in our case 500) weekly simulations using a plain (i.e. not filtered) resampling of the daily variations.

For spot commodity prices

$$S_\ell^{\text{scen}} = \Pi^{\text{rnd1}_\ell(k)} \cdot \Pi^{\text{rnd2}_\ell(k)} \cdot \Pi^{\text{rnd3}_\ell(k)} \cdot \Pi^{\text{rnd4}_\ell(k)} \cdot \Pi^{\text{rnd5}_\ell(k)} \cdot S_{\text{ref}}^h ,$$

for convenience-yield curves (omitting the τ dependence)

$$\begin{aligned} y_\ell^{\text{scen}} &= y^{\text{ref}} + \Delta\gamma^{\text{rnd1}_\ell(k)} + \Delta\gamma^{\text{rnd2}_\ell(k)} \\ &\qquad + \Delta\gamma^{\text{rnd3}_\ell(k)} + \Delta\gamma^{\text{rnd4}_\ell(k)} + \Delta\gamma^{\text{rnd5}_\ell(k)} , \end{aligned}$$

and similarly for interest rates.

Summary of historical simulations

- Instrument price depends deterministically from its risk factors
- In each simulation prices are **at equilibrium** and observed risk factor match simulated ones
- Risk factors are simulated starting from the current quote
- Prices are computed using the simulated risk factors and approximate the instrument distribution of future prices

Deterministic price variations

The price scenarios should not be computed on the reference date t_0, however at the scenario date, i.e. $t_0 + 1\,\text{week}$.

As a consequence we measure the deterministic variation of the value:

$$\Delta P_\ell^{\text{scen}} = P\Big(S_\ell^{\text{scen}}, y_\ell^{\text{scen}}(\tau'), z_\ell^{\text{scen}}(\tau')\Big) - P\Big(S, y(\tau), z(\tau)\Big) \tag{36}$$

with

$$\tau' = T - (t_0 + 1\,\text{week}) \qquad \text{and} \qquad \tau = T - t_0 \tag{37}$$

adding and subtracting the null scenario $P^{\text{null}} = P(S, y(\tau'), z(\tau'))$

$$\Delta P_\ell^{\text{scen}} = \Big(P_\ell^{\text{scen}} - P^{\text{null}}\Big) + \Big(P^{\text{null}} - P^{\text{ref}}\Big) \tag{38}$$

the deterministic variation is the second term on RHS.

Scenarios for a commodity-futures contract

The futures-contract null scenario is given by the implied price at the scenario date

$$F^{\text{null}} = F(S, y(\tau'), z(\tau')) = S \cdot \frac{e^{-y(\tau')\,\tau'}}{e^{-z(\tau')\,\tau'}} \tag{39}$$

The futures simulated scenarios can be computed as

$$F_\ell^{\text{scen}} = S_\ell^{\text{scen}} \cdot \frac{e^{-y_\ell^{\text{scen}}(\tau')\,\tau'}}{e^{-z_\ell^{\text{scen}}(\tau')\,\tau'}}\,. \tag{40}$$

For short maturities $F_\ell^{\text{scen}} \simeq S_\ell^{\text{scen}}$

Scenarios for a commodity futures spread

For $W = 1$, setting $F = F(\tau'_1)$, $R = r_{12}T_{12}$, and $Y = y_{12}T_{12}$ the *null scenario* for a futures-spread contract is given by

$$P^{\text{null}} = F^{\text{null}} \cdot \left[1 - \frac{1+R}{1+Y}\right] \tag{41}$$

The scenarios can be computed as

$$P_i^{\text{scen}} = F_\ell^{\text{scen}} \cdot \left[1 - \frac{1+R_\ell^{\text{scen}}}{1+Y_\ell^{\text{scen}}}\right] . \tag{42}$$

Computation of risk statistics

Given the current bond price P^{ref} and the simulated prices P_ℓ^{scen} we can consider the **distribution** of the N simulated profits and losses,

$$\mathcal{D} = \left\{ P_\ell^{\text{scen}} - P^{\text{ref}} \right\}_{\ell=1,\ldots,N}$$

Risk/Upside	**Percentile**	**Description**
Value-at-Risk	95% and 99%	Worst 5% (or 1%) losses
Exp-Shortfall	95% and 99%	Average of worse than 5% (or 1%) losses
Pot-Upside	95% and 99%	Best 5% (or 1%) gain
Exp-Upside	95% and 99%	Average of better than 5% (or 1%) gains

Risk statistics with the QuantLib addin

First create the statistic accumulator

```
stat = qlStatistics("stat", Values) -> "stat#0001"
```

Then compute the significant risk quantities

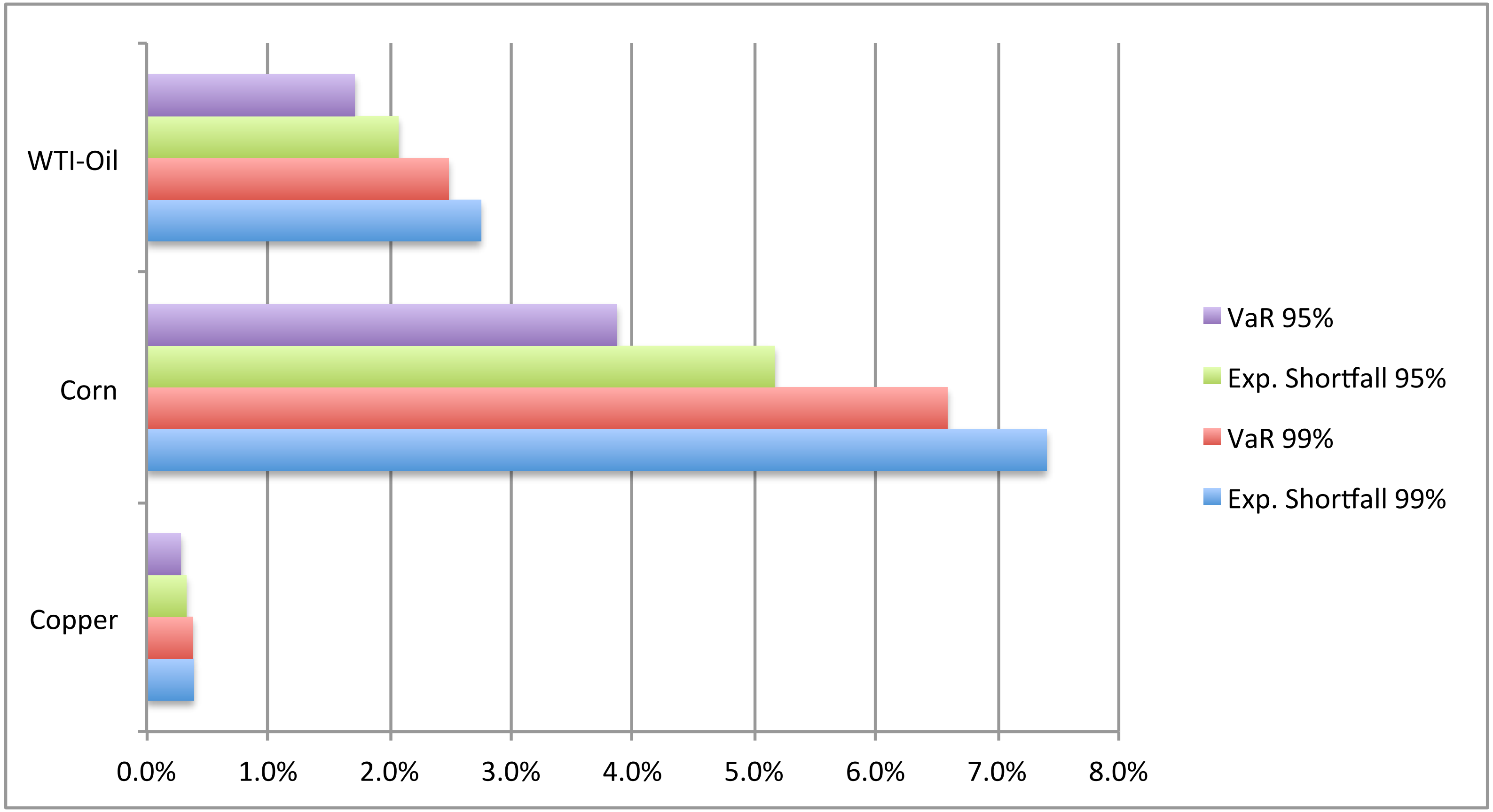
```
var95 = qlStatisticsValueAtRisk("stat", 0.95)
 es95 = qlStatisticsExpectedShortfall("stat", 0.95)
 pu95 = qlStatisticsPotentialUpside("stat", 0.95)
 tp95 = qlStatisticsTopPercentile("stat", 0.95)
```

Risk results for commodity futures spread

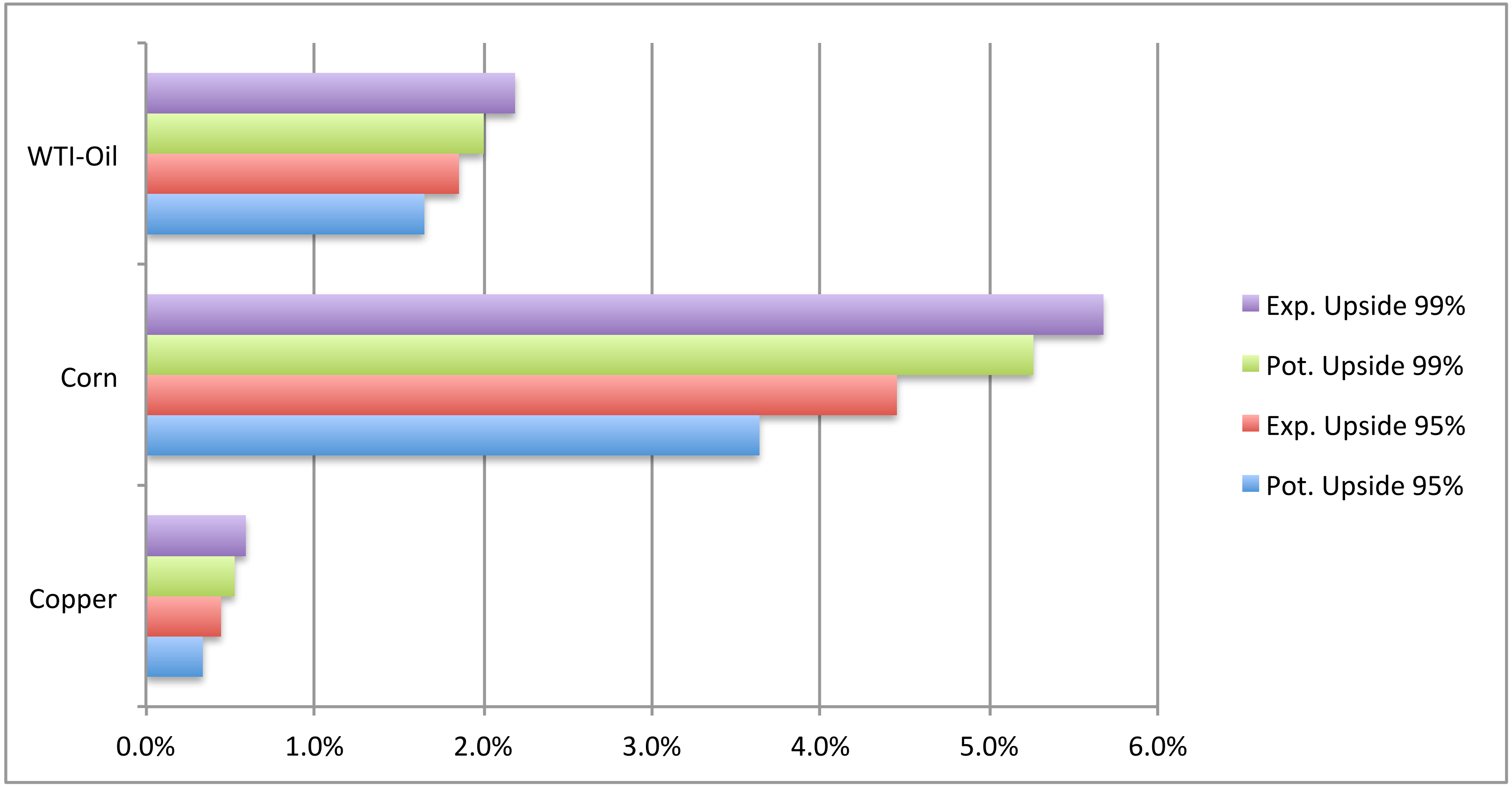
With W so that P=1000 we have the following numerical results

	Copper	**Corn**	**WTI-Oil**
F_{T_1}	8373	629.75	100.64
$F_{T_1} - F_{T_2}$ exposure	1000	1000	1000
Exp. Shortfall 99%	1419.68	956.89	1398.48
VaR 99%	1381.85	851.14	1263.18
Exp. Shortfall 95%	1184.36	667.43	1052.73
VaR 95%	1024.97	499.70	870.37
Pot. Upside 95%	1225.95	469.56	838.82
Exp. Upside 95%	1617.25	574.84	942.53
Pot. Upside 99%	1907.95	679.24	1017.36
Exp. Upside 99%	2153.75	732.96	1111.34

Risk results (as % of $F(T_1)$)



Upside results (as % of $F(T_1)$)



Linear decomposition of scenarios in factors

The spread-contract null pricing function

$$P^{\mathsf{null}} = P(F, R, Y) = F \cdot \left[1 - \frac{1+R}{1+Y}\right], \tag{43}$$

can be linearized to give the price increments as

$$\begin{aligned}\Delta P_i^{\mathsf{scen}} \;&=\; \left(F\,\frac{\partial P}{\partial F}\right)\frac{\Delta F_\ell^{\mathsf{scen}}}{F} + \left(\frac{\partial P}{\partial R}\right)\Delta R_\ell^{\mathsf{scen}} \\ &\quad + \left(\frac{\partial P}{\partial Y}\right)\Delta Y_\ell^{\mathsf{scen}} + \text{Non-Lin}_\ell^{\mathsf{scen}}\,.\end{aligned} \tag{44}$$

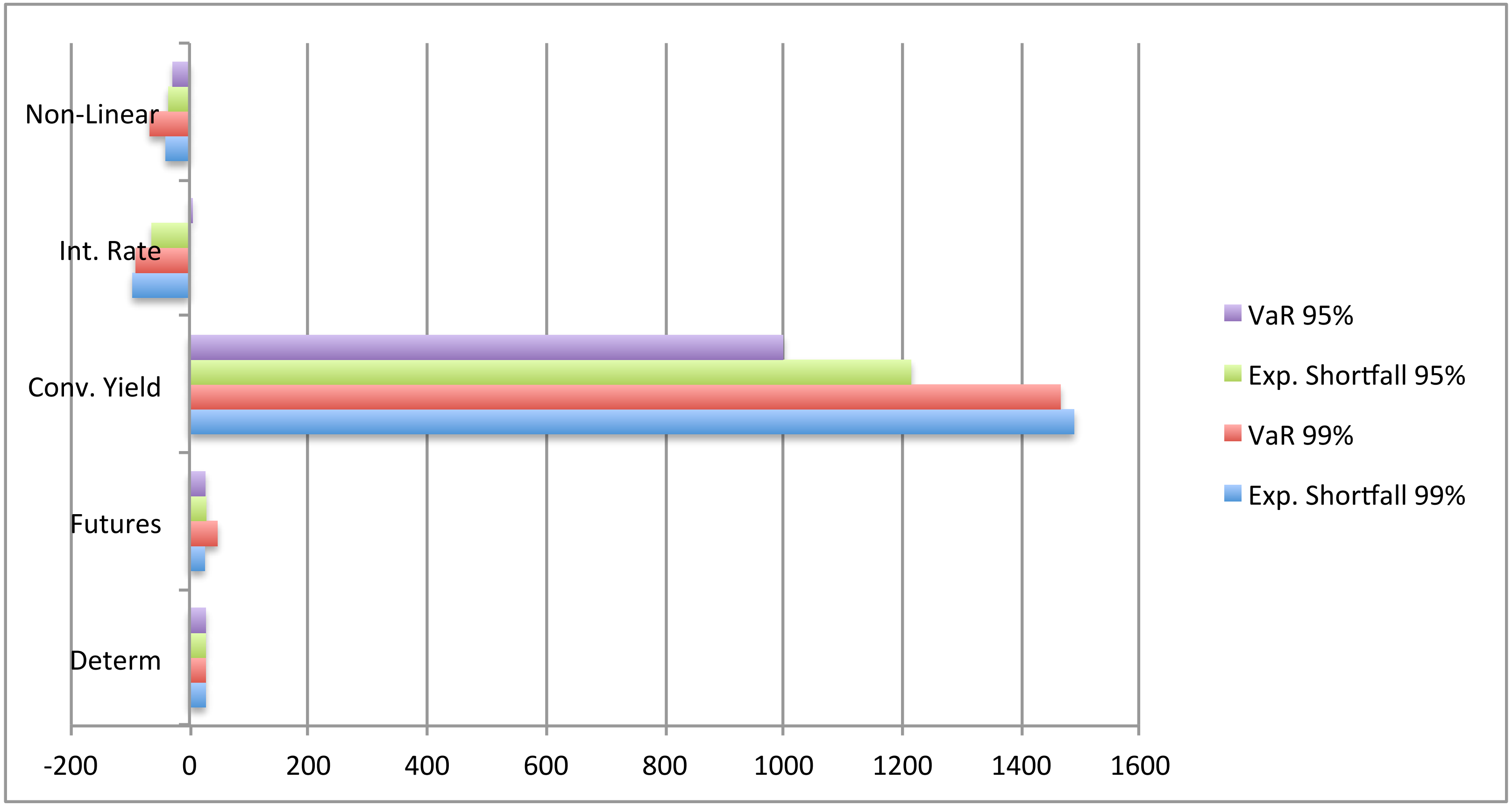
Decomposition of risk in factors

- After computing the risk figures we trace back the scenarios that give rise to risk
- Note that the risk figures are additive in those scenarios
- Use the linear scenario decomposition on those scenarios and sum up each term to compute the risk decomposition

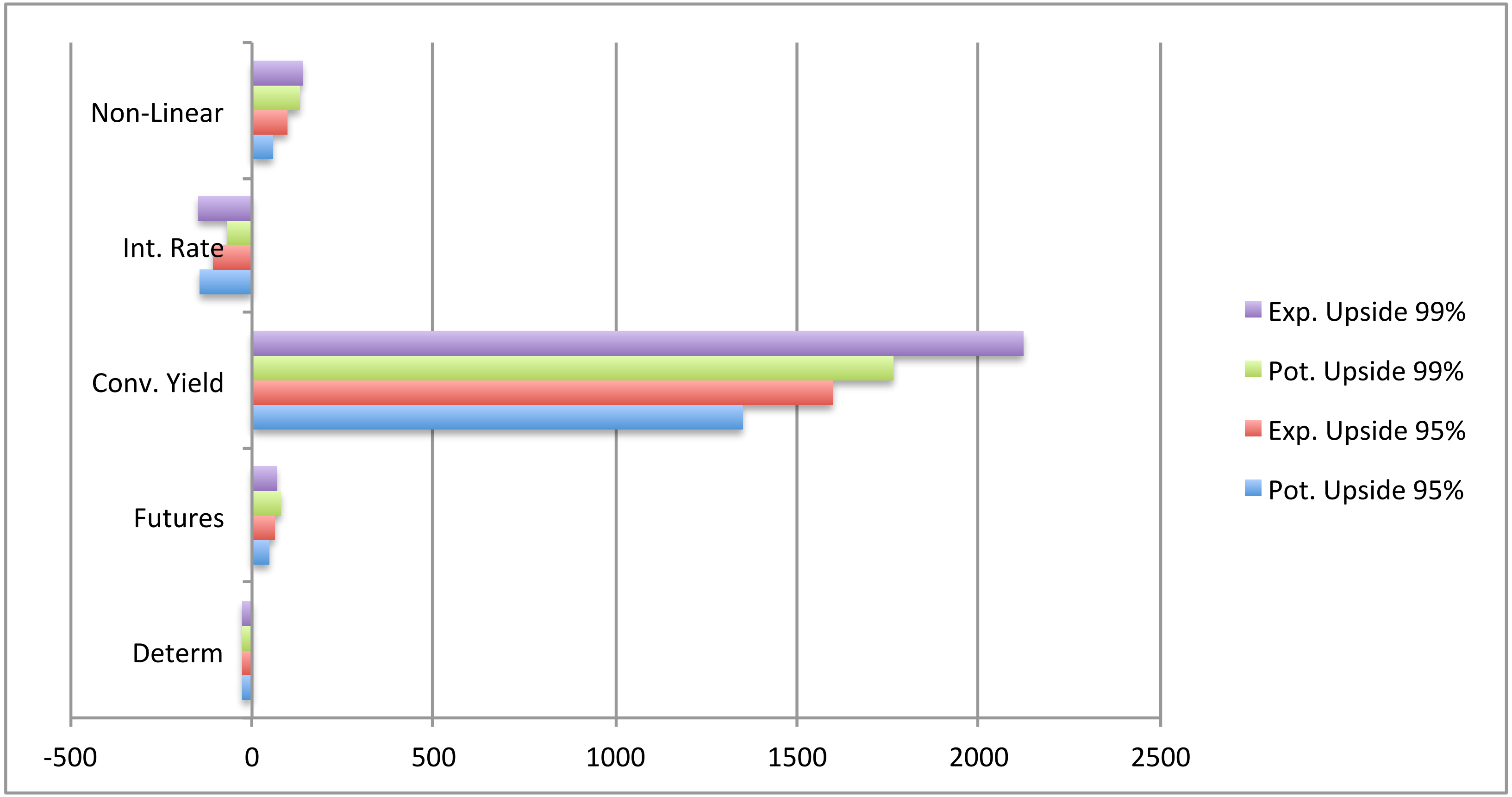
Decomposition results for Copper

P=1000	**Det**	**Stoc.**	**Fut**	**Conv.Y**	**Rate**	**non-lin**
Exp. Shortfall 99%	27.12	1377.86	25.58	1488.75	-96.12	-40.34
VaR 99%	27.12	1354.73	47.03	1465.66	-90.76	-67.20
Exp. Shortfall 95%	27.12	1143.35	28.29	1214.30	-63.48	-35.76
VaR 95%	27.12	997.85	26.23	999.19	0.51	-28.07
Pot. Upside 95%	-27.12	1309.27	46.94	1349.85	-144.36	56.83
Exp. Upside 95%	-27.12	1650.57	62.25	1597.69	-106.85	97.48
Pot. Upside 99%	-27.12	1906.48	78.94	1763.79	-67.11	130.86
Exp. Upside 99%	-27.12	2180.87	67.94	2122.07	-147.59	138.45

Risk decomposition for Copper



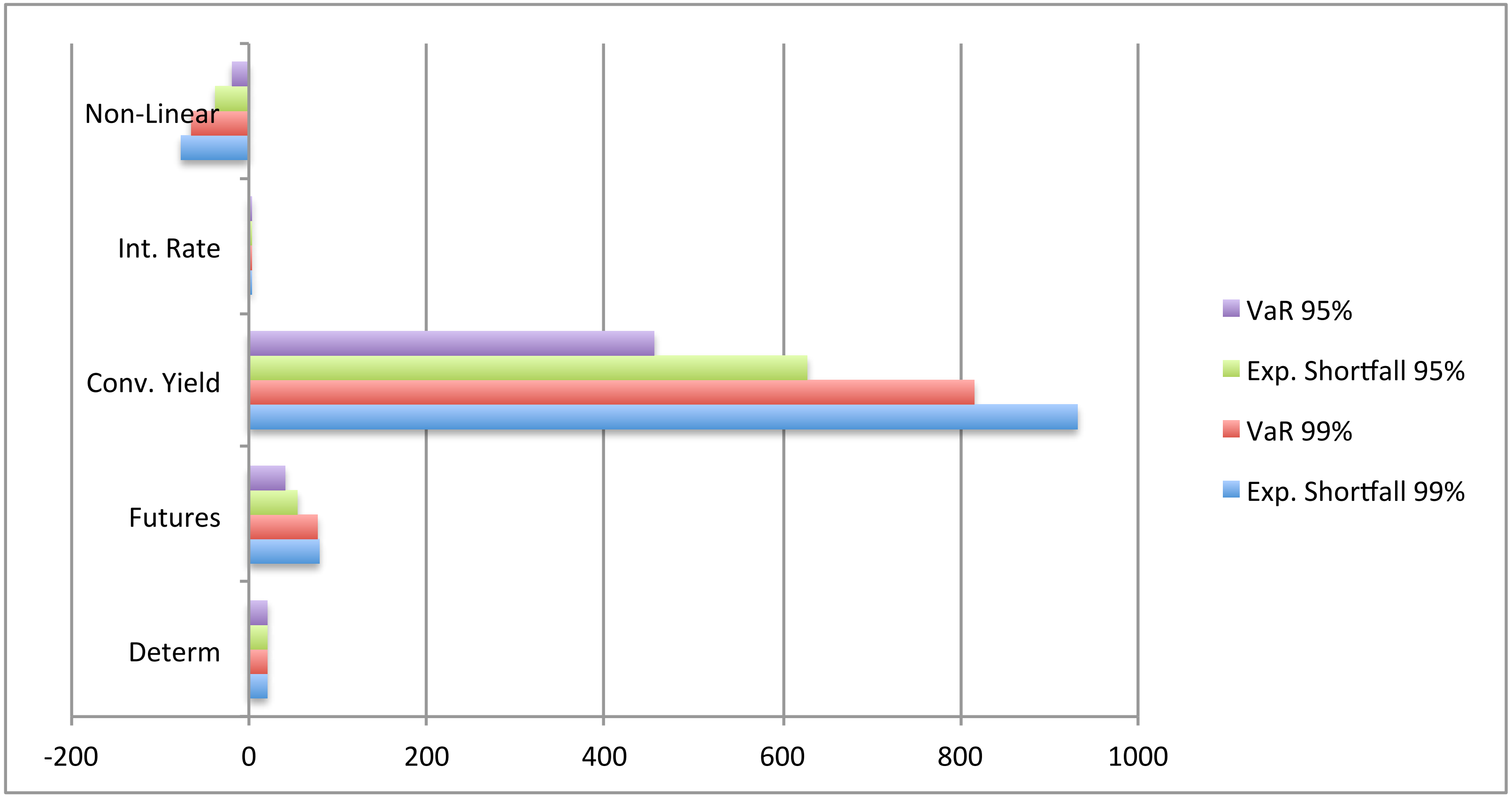
Upside decomposition for Copper



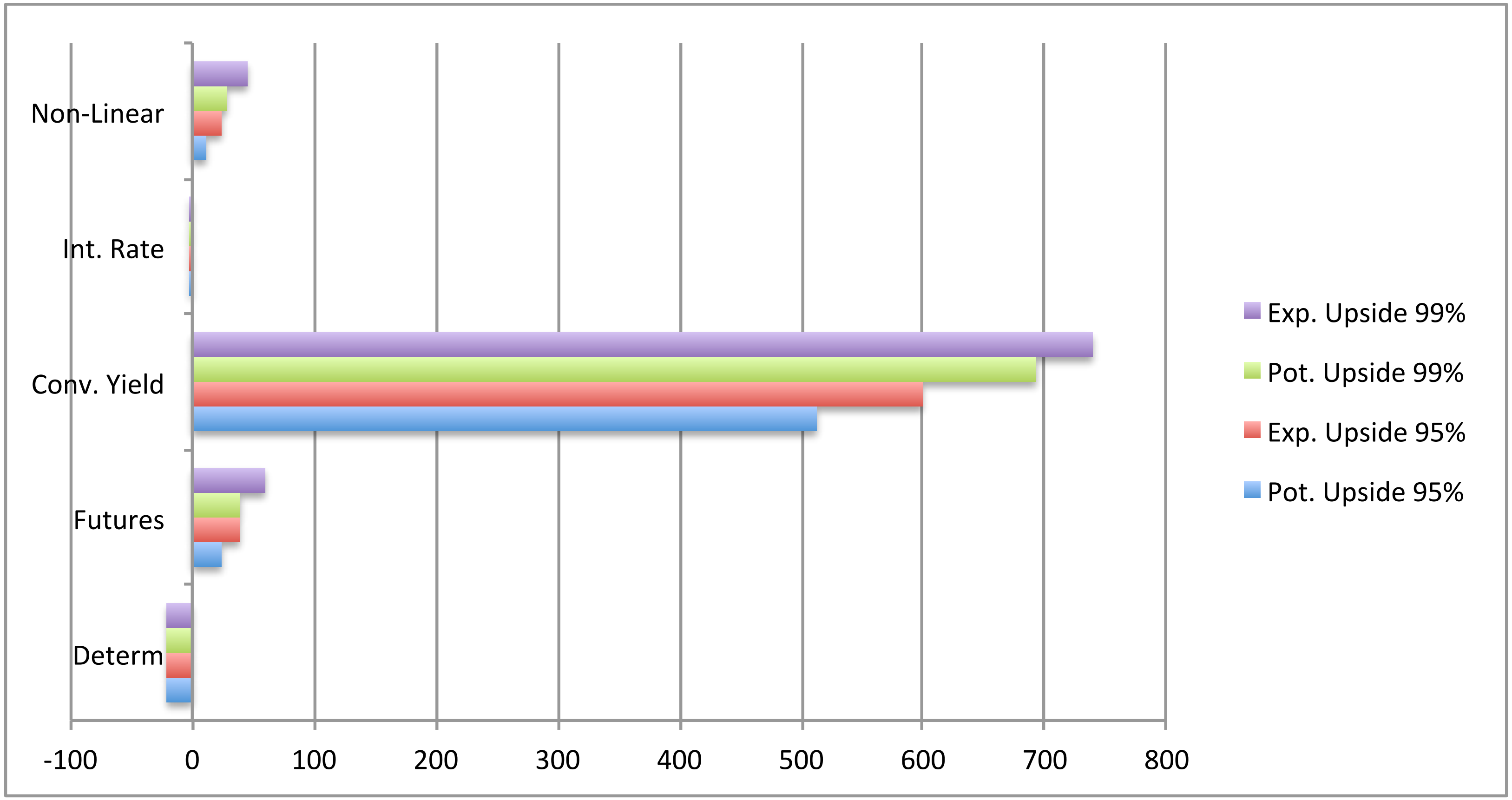
Decomposition results for Corn

P=1000	**Det**	**Stoc.**	**Fut**	**Conv.Y**	**Rate**	**non-lin**
Exp. Shortfall 99%	20.92	935.97	79.17	931.95	1.08	-76.23
VaR 99%	20.92	830.22	77.23	815.62	2.00	-64.62
Exp. Shortfall 95%	20.92	646.51	54.60	627.84	1.77	-37.70
VaR 95%	20.92	478.79	40.73	455.89	1.18	-19.02
Pot. Upside 95%	-20.92	506.16	24.09	512.64	-2.06	11.44
Exp. Upside 95%	-20.92	634.35	38.89	599.61	-1.93	23.94
Pot. Upside 99%	-20.92	738.33	39.20	692.71	-0.91	28.25
Exp. Upside 99%	-20.92	823.53	59.74	739.27	-0.02	45.46

Risk decomposition for Corn



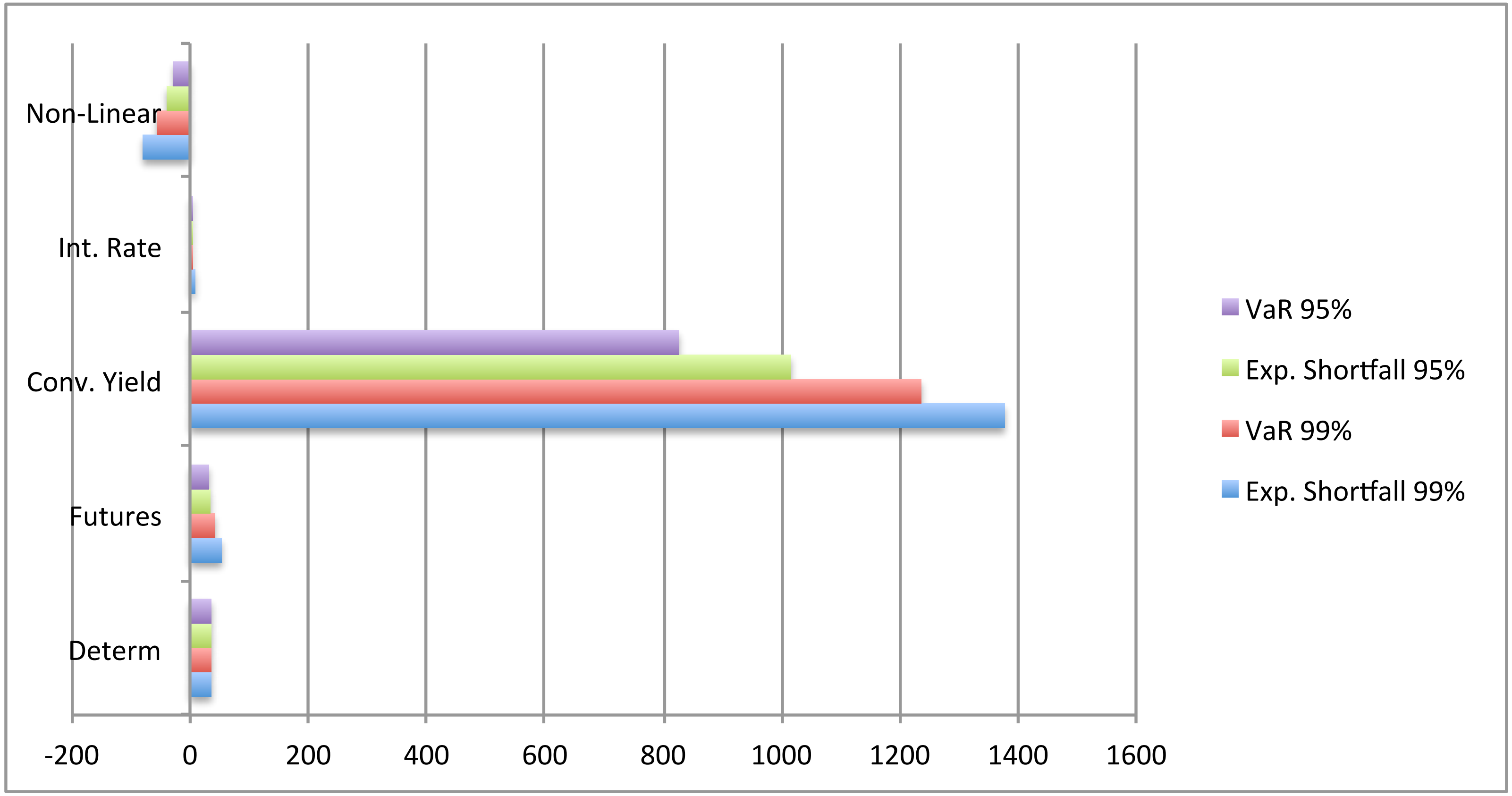
Upside decomposition for Corn



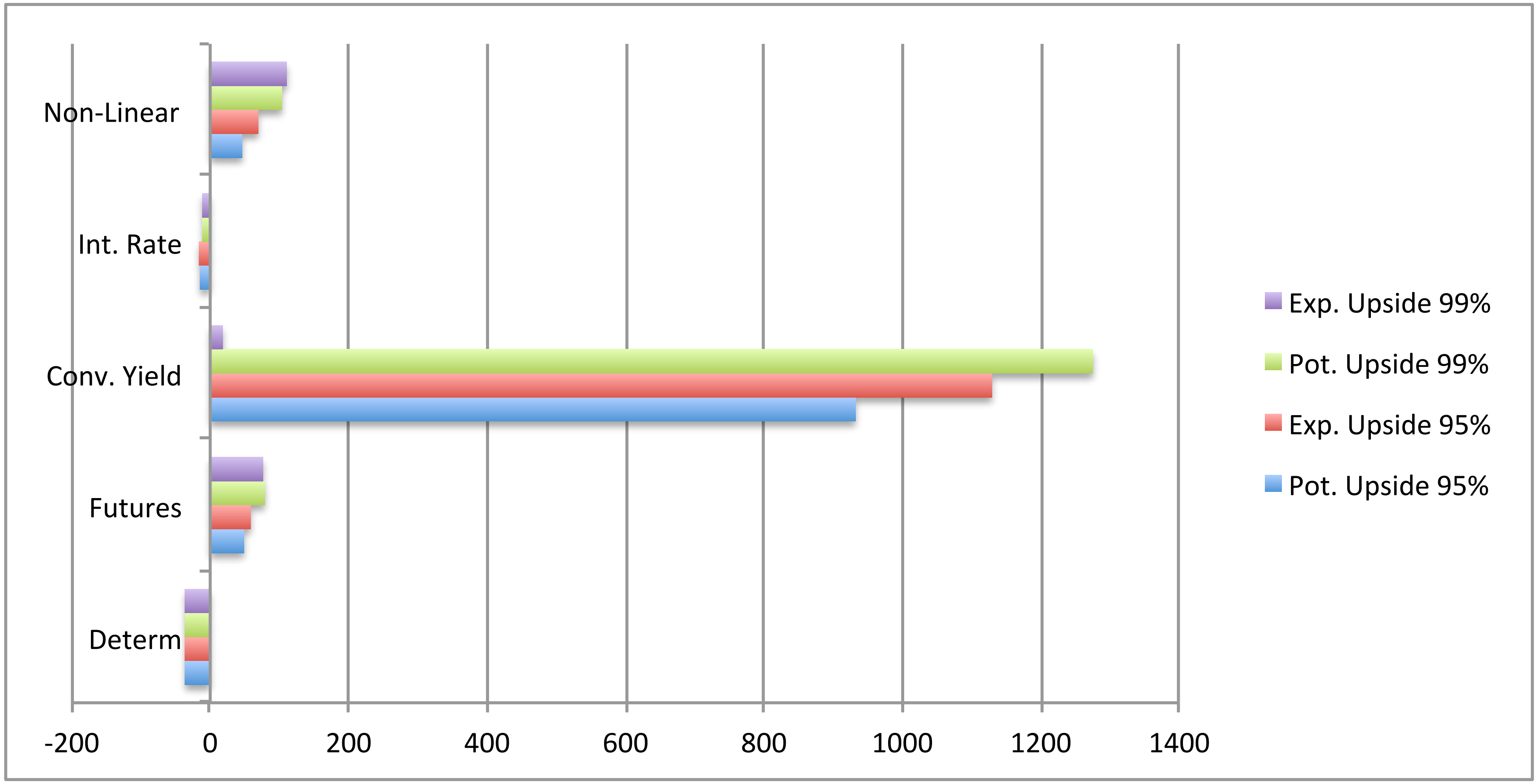
Decomposition results for WTI Oil

P=1000	**Det**	**Stoc.**	**Fut**	**Conv.Y**	**Rate**	**non-lin**
Exp. Shortfall 99%	36.32	1360.30	53.85	1377.67	8.78	-80.00
VaR 99%	36.32	1226.86	42.41	1236.78	3.51	-55.83
Exp. Shortfall 95%	36.32	1016.41	34.57	1016.36	4.93	-39.45
VaR 95%	36.32	834.05	32.15	826.15	3.51	-27.76
Pot. Upside 95%	-36.32	935.77	48.67	932.34	-14.97	46.33
Exp. Upside 95%	-36.32	1196.75	58.59	1129.41	-15.90	69.60
Pot. Upside 99%	-36.32	1409.82	78.62	1274.84	-11.06	103.74
Exp. Upside 99%	-36.32	1552.79	76.23	18.22	-11.50	110.70

Risk decomposition for WTI Oil



Upside decomposition for WTI Oil



Summary

- Linear commodity derivatives are used to hedge/take-risk in future commodity prices
- Linear derivatives PV depend on commodity prices, convenience yields, and interest rates
- Short-maturity futures provide good proxies for spot prices
- Longer futures contracts increasingly depend on the convenience-yield curve
- We simulated weekly variations of commodity futures spreads
- For the observed commodities risk and upside mostly arises from the contribution of convenience yields

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