Integrating default risk in the historical simulation model

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Abstract

We describe a new model for generating credit risk scenarios in the *StatPro Simulation Model*. Starting from the historical series of asset swap indices grouped by sector, currency, and rating, we can derive a number of equivalent time series for the zero volatility spreads(or z-spreads). The current credit risk of an asset is modeled using the z-spread so computed. In order to simulate the change of credit risk from one day to another we employ two different procedures depending on the availability of a default probability structure for the given issuer. The first method, namely the *static method*, relies purely on the issuer rating for obtaining the spread scenarios. The second method, namely the *dynamic method*, interpolates a *fractional rating* to accurately position the given issuer between ratings. Interestingly, the dynamic method gives an immediate response to event risk since the approach reflects the risk that is embedded in the market-quoted default probability structure.

1 Introduction

In another paper, see reference [3], we describe how to compute the expected distribution for a financial instrument whose value is known in terms of some risk factors. In summary, consider a financial instrument π for which it is possible to compute the price at the current time t as a function of n risk factors r_t^1, ..., r_t^n; possibly depending on K parameters X^1, ...,X^K,

$$\pi_t = f_\pi \left(t;\, X^1, \ldots, X^K;\, r_t^1, \ldots, r_t^n \right) \,. \tag{1}$$

Consider now the market value of π at the future time T. Conditional to the realization of a given scenario i(for i=1, ..., N), each risk factor assumes a value r_i^k. Following the procedure of reference [3] the scenarios π_i for the instrument price at the future time T are given by,

$$\pi_i = f_\pi \left(T;\, X^1, \ldots, X^K;\, r_i^1, \ldots, r_i^n \right) \,. \tag{2}$$

It is necessary, therefore, to know the expected distribution for all the underlying risk factors in order to compute the expected distribution of an asset. In this paper we describe in detail how to include the credit risk factor in the simulated scenarios.

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2 Pricing an instrument in presence of credit risk

In this section we discuss the pricing of an instrument in the presence of credit risk. First, we consider the simple case of a zero-coupon bond because of its simplicity. Later the procedure is generalized to a generic asset. Once the pricing procedure has been established it is straightforward to adapt the general procedure of reference [3] to compute the simulated scenarios that include credit risk.

2.1 Price for a zero-coupon bond in presence of credit risk

Consider a zero-coupon bond issued by a risky entity with associated default-probability term structure p and recovery ratio R. As shown in reference [2] the bond price at the current time t is given by

$$P_t^{\mathrm{zc}} = 100 \ \{[1 - p(t, t_m)] + R\, p(t, t_m)\,\}\, D_r(t_s, t_m;\ f_t^1,\, \ldots,\, f_t^n)\,, \tag{3}$$

where t_s is the bond settlement date[1], t_m is the bond maturity expressed in years, and D_r is the risk-free discount factor. This expression simply states that the expected redemption payment at time t_m is given by the sum of the redemption in case of no default, i.e. 100, multiplied by the probability of survival $1 - p(t, t_m)$, plus the redemption in case of default, i.e. $100\,R$, multiplied by the probability of default $p(t, t_m)$. Notice that the probability $p(t, t_m)$ is computed in the risk-neutral world and can be bootstrapped, e.g., from the market data of a string of credit default swaps(see, again, reference [2]).

The equivalent of expression (2) for the bond scenarios is given by,

$$P_i^{\mathrm{zc}} = 100 \ \{[1 - p_i(T, t_m)] + R\, p_i(T, t_m)\,\}\, D_r(T_s, t_m;\ f_i^1,\, \ldots,\, f_i^n)\,, \tag{4}$$

where T_s is the time of bond settlement, for each k, $\{f_i^k\}$ is the expected distribution of the k-th forward rate at time T, and, for each i, p_i is the the risk-neutral probability of default conditional to the realization of scenario i. Therefore, in order to properly account for credit risk in the distribution $\{P_i^{\mathrm{zc}}\}$ it is necessary to generate the simulated scenarios for the default probability $p(T, t_m)$. In order to generate this distribution it is convenient to express equation (3) in terms of the zero-volatility spread defined below.

Zero volatility spread The zero-volatility spread, or simply z-spread, is defined as the continuously-compounded spread z that should be applied on the Libor curve in order to price a bond consistently with market price. More details on the z-spread for a fixed-rate coupon bond are given in the paper [2]. For a zero-coupon bond it is straightforward to compute the z-spread z as the number such that,

$$P_t^{\mathrm{zc}} = 100\, e^{-z\,(t_m - t_s)} D_{\mathrm{L}}(t_s, t_m)\ , \tag{5}$$

where $D_{\mathrm{L}}(t_1, t_2)$ is the discount from time t_1 to t_2 on the Libor curve and P_t^{zc} is the bond price either obtained from equation (3) or observed on the market. From this equation it follows that,

$$\frac{P_t^{\mathrm{zc}}}{100\, D_{\mathrm{L}}(t_m)} = \frac{1}{e^{z\,(t_m - t_s)}}\ , \tag{6}$$

and therefore,

$$z = \frac{1}{(t_m - t_s)} \log\left[\frac{100\, D_{\mathrm{L}}(t_m)}{P_t^{\mathrm{zc}}}\right]\ . \tag{7}$$

Note that the z-spread could be a negative number and depends on the bond maturity.

[1]Note that t_s is a function of the current time t.

Scenarios for credit risk Expressing explicitly the discount Libor curve $D_{\rm L}$ in terms of the forward rates $f_t^1, \ldots, f_t^n$ we can write the zero-coupon bond price as,

$$P_t^{\rm zc} = 100\; e^{-z_t\,(t_m - t_s)} D_{\rm L}(t_s, t_m;\, f_t^1, \ldots, f_t^n)\,, \tag{8}$$

where z_t is the z-spread at the current time t. Consider this expression as the starting point to generate the simulations and define the function f_π of equation (1) from it. Given the forward rate distributions $\{\, f_i^1\}$, ..., $\{\, f_i^n\}$ equation (2) for the risk scenarios $P_i^{\rm zc}$'s becomes,

$$P_i^{\rm zc} = 100\; e^{-z_i(t_m - T_s)} D_{\rm L}(T_s, t_m;\, f_i^1, \ldots, f_i^n)\,, \tag{9}$$

for an appropriate choice of the z-spread distribution $\{z_i\}$. The goal of the present paper is to define the scenarios z_i's so that the price scenarios incorporate the change of credit risk.

Defaulted scenarios Given a fixed recovery ratio R, regardless of the choice of the z-spread distribution $\{z_i\}$, the worst scenario case happens when the issuer defaults for sure, i.e. with probability one, in the risk neutral (probability) space. This condition is met for all the values of the index i such that scenarios $p_i(T, t_m)$=1. Therefore, by comparing equations (4) and (9) it follows that all the scenarios with index i such that

$$R \geq e^{-z_i(t_m - T_s)}, \tag{10}$$

should be considered as defaulted scenarios. Similarly, any simulated z-spread z_i such that,

$$z_i \geq z_D, \tag{11}$$

where

$$z_D = \frac{-\log R}{(t_m - T_s)}, \tag{12}$$

can be assumed to be a default scenario. In practice no precise value for the recovery ratio can be established since R cannot be measured exactly before default, however, it may still make sense to assume that defaults are reached when condition (11) is satisfied for some value of z_D.

2.2 Price of a generic asset in presence of credit risk

The pricing function for a zero-coupon bond given by equation (8) can be generalized to the pricing function of a generic product with a more complex structure of cash flows. Consider an instrument that pays K coupons C_j, for j=1, ..., K, depending on Q parameters $X^1, \ldots, X^Q$, the interest rates $f_t^1, \ldots, f_t^n$, and, possibly, M other risk factors[2] denoted by $r_t^1, \ldots, r_t^M$. The arbitrage-free price can therefore be computed as

$$P_t = \sum_{j=1}^{K}\; e^{-z_t\,(t_j - t_s)} D_{\rm L}(t_s, t_j;\; f_t^1, \ldots,\; f_t^n)\, C_j\left(X^1, \ldots, X^Q;\, f_t^1, \ldots, f_t^n, r_t^1, \ldots, r_t^M\right)\,, \tag{13}$$

where z_t is the z-spread at the current time t. Note that the extra risk factors can be any of the types described in reference [3]. Given the forward rate distributions $\{f_i^1\}, \ldots, \{f_i^n\}$ and the distributions for the other risk factors $\{r_i^1\}, \ldots, \{r_i^M\}$ the simulated scenarios P_i's can be computed from equation (2) as follows

[2]Note that, in certain cases, such as convertible bonds, the z-spread itself could be part of these risk factors.

Sectors	Subsectors		
all sectors	all subsectors		
financial	all financial		
	insurance	finance	bank
industrial	all industrial		
	consumer products	health care	media
	gaming and lodging	manufacturing	energy
	basic industries	technology	retail
	transportation	telecom	property
public	all public		
	agency	provincial	supranational
	sovereign	state guaranteed	
utility	all utility		
	electric	reg transp	gas pipelines

Table 1: List of sectors and subsectors available for different issuers

$$P_i = \sum_{j=1}^{K} e^{-z_i\,(t_j - T_s)} D_{\mathrm{L}}(t_s, t_j;\; f_i^1, \ldots,\; f_i^n)\, C_j\left(X^1, \ldots, X^Q;\, f_i^1, \ldots, f_i^n, r_i^1, \ldots, r_i^M\right)\,. \tag{14}$$

Notice how the z-spread distribution $\{z_i\}$ can be used to compute the scenario distribution $\{P_i\}$ for any asset depending also on credit risk. Next section describes how such a distribution can be obtained for a rated issuer, section 4 describes another way to compute spread distribution that matches expectations of credit-quality changes.

3 Static simulation of credit-spread scenarios

As shown in the previous section it is convenient to compute credit risk as z-spread scenarios. The present section computes z-spread distribution $\{z_i\}$ for bonds whose issuer has a given rating. The methodology for generating the simulated z-spread scenarios for a given issuer is different depending on the availability or not of a default-probability term structure for the issuer of the instrument under consideration. The default-probability term structure can, for example, be obtained when one or more credit default swaps are available for the issuer. See reference [2] for more details on the subject.

Issuer sectors and rating One of the big challenges of a credit-risk model is the collection of good credit-related data usable with a large number of different issuers. Even a sophisticated model is useless without reliable data since it cannot be applied to the issuers present in the markets. StatPro relies on a very broad data set composed by the daily, or weekly, fixings of asset-swap indices for a number of ratings, covering a number of sectors and subsectors(see Table 1), for issues in the major currencies. For simplicity, we will use the terminology *sector/rating/currency* to identify the group of issues with the same currency, rating, sector and subsector.

3.1 Credit scenarios for a specific maturity and rating

Consider a generic issuer belonging to a specific *sector/rating/currency* group, for example *industrial/basic industries*, AA, EUR, together with its historical time series of the asset-swap spreads.

Name	Maturity
y_1	1 year
y_2	3 years
y_3	5 years
y_4	10 years
y_5	20 years
y_6	30 years
y_7	45 years
y_8	60 years

Table 2: Example of maturity buckets for dummy bonds used to convert the asset-swap spreads into z-spreads.

As shown in the previous section, the z-spread on a certain interest-rate curve depends on the maturity of the underlying bond. The historical data from the asset-swap spreads is, therefore, converted in equivalent z-spread series with the help of some *dummy*, or *synthetic*, bonds that span all available maturities. Define a number of maturities y_m for m=1, ..., n_m, such as those listed in Table 2. For each maturity y_m, define a dummy fixed-rate coupon bond B^m, with maturity y_m paying a fixed coupon equal to the average coupon paid by the market at that maturity for that currency. Given an asset-swap spread A_t, for each dummy bond B^m, compute the bond price B_t^m at the current time t and the corresponding z-spread[3] $\hat{z}_t^m$. Notice that the asset swap spread A_t does not need to be, and in general is not, the same for all maturities. The z-spread $\hat{z}_t^m$ so computed should be used in equation (13) for the pricing of the financial instrument with maturity m, i.e.,

$$z_t = z_t^m \,. \tag{15}$$

To obtain the z-spread scenarios at a certain maturity m, compute the z-spreads $\hat{z}_i^m$'s for each past date i using the historical asset-swap spreads A_i's. Note that the historical dummy-bond prices are computed, for each day, with the interest-rate term structure for that day. In this way compute and store the historical z-spread $\hat{z}_i^m$, for each day i and maturity m.

The computation of the z-spread scenarios z_i's can be performed using the historical method for absolute variations as shown in reference [3]: compute first the historical differences in spread ξ_i^m's from one day to another,

$$\xi_i^m \; = \; \hat{z}_i^m - \hat{z}_{i-1}^m, \tag{16}$$

and apply this difference ξ_i^m to the current z-spread z_t, i.e.,

$$z_i = z_t + \xi_i^m. \tag{17}$$

The scenario distribution $\{z_i\}$ thus computed should be used in equation (14) to compute the instrument expected distribution.

3.2 Example: spread scenarios for an investment-grade issuer

To understand the details of the simulation process, consider a very simple example of spread-scenario computation for a generic issuer with an investment-grade rating: AAA, AA, A, or BBB. To simplify the discussions that follow, unless otherwise specified, the term *spread* will be used instead of the more technical term *z-spread*. For simplicity, it will be also assumed that the expected scenarios are computed for a specific bond with exactly the same maturity as a dummy bond, e.g.

[3]Note that $\hat{z}_i$ denotes the historical value, while z_i the simulated value of the z-spread.

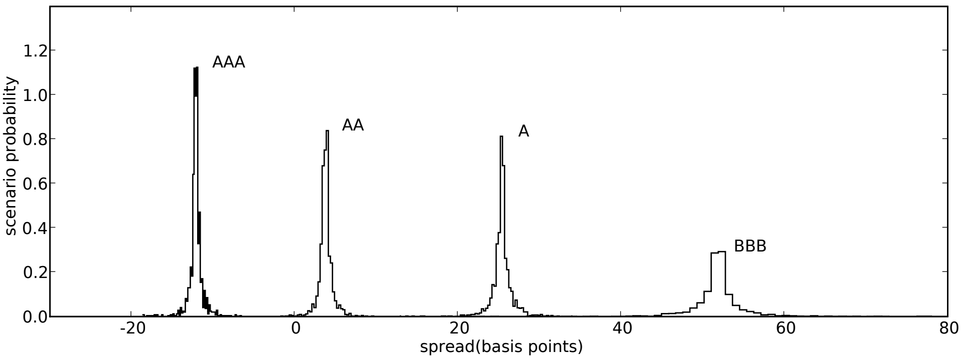

Figure 1: The simulated distributions $\{z_i^4\}$ for currency EUR, for the investment-grade ratings AAA, AA, A, BBB; for *all-sectors/all-subsectors*. From the plot it is evident how the distributions are leptokurtic.

y_m=y_4=10 years. Furthermore, to make the example clearer, we will use the asset-swap spreads for the currency EUR relative to the category *all-sectors/all-subsectors*.

Let A_t be the current asset-swap spread and, let A_i, for i=1, ..., N, be the historical values for the asset-swap spreads. Furthermore, let B^4 be a fixed-rate coupon bond maturing in ten years, with a coupon equal to the average coupon of issues with the same risk profile in the same currency. Using the techniques of reference [2], it is possible to compute the z-spread z_t^4 for the bond B^4 assuming an asset-swap spread A_t. Finally, consider the collection of all the spreads $\hat{z}_i^4$'s computed historically in the same way at each date i. Note that in order to compute each $\hat{z}_i^4$ the historical interest-rate curve was used.

At this point, expression (17) allows the computation of the z-spread scenarios

$$z_i^4 = z_t^4 + \xi_i^4 \,, \tag{18}$$

where, according to definition (16),

$$\xi_i^4 = \hat{z}_i^4 - \hat{z}_{i-1}^4 \,. \tag{19}$$

The probability distributions of $\{z_i^4\}$ so computed are plotted in Figure 1 for the investment-grade ratings AAA, AA, A, and BBB, at the end of the year 2006. From the same Figure few remarks are in order:

- As widely expected, the location of the reference spread z_t^4 increases with a decreasing rating.
- The probability distributions have tall peeks around the reference spread: the distributions are qualitatively leptokurtic.
- As the rating decreases the probability distributions widens and their peaks lowers.

In order to quantitatively substantiate these observations, in Table 3 we also show the values, in basis points, for the reference spread z_t^4, the standard deviation σ, and kurtosis κ of the distributions for z_i^4.

Notice how the standard deviation σ increases from 0.867 basis points of the AAA to the 3.822 basis points of the BBB. Recall that these values were computed from the daily variations and cannot be compared directly with the annual probability of default usually mentioned in literature.

Since the kurtosis κ is large and positive for all ratings, we can conclude that the z-spread distribution are far from being Gaussian and show the well known fat-tail phenomenon. Given these numbers for κ it may seem unreasonable to model the spread distributions as normal.

3.3 Credit risk scenarios for a given rating and all maturities

As it was already anticipated in the previous section, StatPro uses asset-swap-spread indices as building blocks for simulating the credit component of the scenario distribution. The asset-swap data is then transformed into z-spreads for a discrete number of maturities. Given a rating, a sector, and a currency we have also described how scenarios for z-spread $\{z_i^m\}$ can be computed for a certain maturity m.

Consider the computation of scenarios for a bond, or another type of instrument, with a maturity y different from y_m for any m and let m be such that $y_m \leq y < y_{m+1}$. The idea is to build the simulation scenarios as a *mix* of the scenarios at maturities y_m and y_{m+1}. To this purpose define the maturity fraction γ as

$$\gamma = \frac{Y(0,y) - Y(0,y_{m+1})}{Y(0,y_m) - Y(0,y_{m+1})}, \tag{20}$$

where $Y(t_1,t_2)$ is the year fraction between t_1 and t_2 for some given day count convention Y, for example Actual/Actual(ISDA)[4]. The reference z-spread z_t used to compute the reference value of the instrument at the current time t is then defined as

$$z_t = \gamma\, z_r^m + (1-\gamma) z_r^{m+1}. \tag{21}$$

Since we are interpolating the reference spread between two different maturities, it makes sense to do the same for the spread scenarios z_i's as well, thus,

$$z_i = z_t + \gamma \xi_i^m + (1-\gamma)\xi_i^{m+1}, \tag{22}$$

where $\{\xi_i^m\}$ are defined by (17) for both m and m+1.

Note It is possible that y is earlier than the first dummy maturity y_1 or after the last one, i.e. y_{n_m}, in these cases we define

$$z_t = z_t^m \tag{23}$$

and

$$z_i = z_t + \xi_i^m, \tag{24}$$

with m=1 in the former case, and m=n_m in the latter case.

Among all the possible interpolation methods that can be applied in equation (22), we choose the linear one because of its simplicity. We leave to future works the investigation on other types of interpolation.

Summary of the method Let us summarize the computation of credit-spread scenarios for issuers for whom a precise term structure for the default probability is not known. Consider a generic financial instrument issued by the reference credit entity. Assign to it a specific asset-swap index according to its sector, instrument currency, and rating. From the same asset-swap class compute the spread for different maturities according to the synthetic bonds. The z-spreads are computed for each maturity for the current time and historically in the past. Given a specific instrument, its reference z-spread and the scenarios are then linearly interpolated between the two dummy maturities containing the instrument maturity to obtain the z-spread scenarios.

[4] Again, reference [1] gives more details on year fractions ad day-count conventions.

	z_t^4	σ	κ
AAA	-12.0	0.867	14.734
AA	3.8	0.943	20.910
A	25.9	1.346	13.949
BBB	52.9	3.822	21.578

Table 3: Values, in basis points, for the reference spread z_t^4, the standard deviation σ, and the kurtosis κ, for the simulated distribution $\{z_i^4\}$ for currency EUR, *all-sector/all-subsectors*, for the same ratings as Figure 1.

4 Dynamic simulation of credit-spread scenarios

In this section we show how to extend the computation of credit-spread scenarios to include changes in market default expectations for particular issuers for whom a risk-neutral term structure for the default probability is available. For simplicity, the focus will be on a bond but the procedure can be applied for any complex product that can be priced using a discount curve as shown in subsection 2.2.

4.1 Historical simulations of credit spreads

Nowadays the credit-derivative market is mature enough to cover a wide variety of names. In particular, it is possible to find credit-default swaps for all the world's major names(sometimes even if no bonds are issued by these names). As shown in reference [2], the existence of one or more credit-default swaps allows the construction of a term structure for the default probability and a precise computation of the z-spread for each given bond. Therefore, the exact spread z_t can be used in expression (13) to compute the current price P_t. In this way the market expectations of default at time t are correctly accounted for. Also, the price P_t obtained in this way should be much closer to the real market price than that obtained using a z-spread obtained from a *sector/rating/currency* group.

In order to compute the expected distribution $\{z_i\}$ required by equation (14), we apply the historical-simulation method described in reference [3]: the historical values of the individual issuers' z-spreads[5] $\tilde{z}_i$ is collected for each past date i and the scenarios are computed as

$$z_i = z_t + \tilde{z}_i - \tilde{z}_{i-1}. \tag{25}$$

Using this approach the instrument price is computed in sync with market default expectations. However, as shown by equation (25), the new scenarios containing information about the changes of the credit risk enter into the spread series only gradually and time is needed before the changes are adequately propagated through the credit-spread distribution. Furthermore, some risk figures, like expected shortfall, will react more quickly to the change in credit risk, VaR will react less quickly; to see the changes in measures such as the volatility it takes even longer. Given these shortcomings, for the reminder of this section, we develop a different method to compute spread scenarios.

[5]Note that the symbols $\tilde{z}_i$'s denote the historical z-spreads computed from the curve of credit-default swaps for the specific issuer and should not be confused with the historical z-spread coming from the asset-swap indices denoted by $\hat{z}_i^m$.

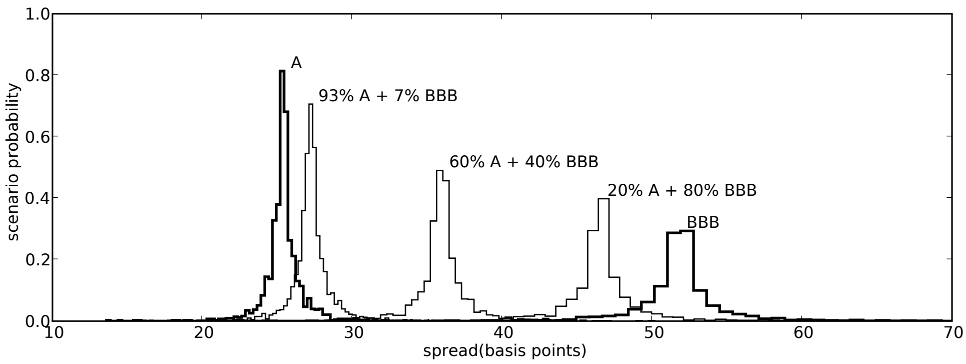

Figure 2: Plot of the simulated distributions $\{z_i^4\}$ for currency EUR, for *all-sectors/all-subsectors,* for the investment-grade ratings A and BBB(thick solid lines). The thin solid lines show the expected distributions obtained from definition (30) for β^4=93%, 60%, 20%.

4.2 Spread simulations with fractional rating

Consider a specific bond belonging to a certain *sector/rating/currency* group. If we were to apply the static method of section 3, then we would have a change in the credit-spread distribution only when the rating of an issuer changes. In this case the scenario distribution would change suddenly and widens, or narrows, abruptly. It is indeed expected that the distribution of a BBB is riskier than that of an A, however we would like to see a smooth transition from one to the other. Furthermore, the rating given by the agencies are only labels: what really matters is the *market view* of a certain name. It would be interesting, in this respect, if we can have a set of continuous ratings that change from one rating to another, and a rating given each day to any name. Unfortunately, such a rating agency does not exist; however, we will show how it is possible to define a fractional rating for an issuer so that its spread distribution seamlessly changes in shape, e.g., from that of an A rated issuer to that of a BBB rated one.

Table 3 shows the average spread of an investment-grade issuer belonging to *all-sector/all-subsector* group, for issues in currency EUR. For some issuers, i.e. those for whom a default probability can be bootstrapped, a precise value for that spread can be computed. However, in general, few(if any) of the issuers will have a spread that matches exactly one of those shown in Table 3. Since the credit spread associated with an issuer is a measure of the expected defaults(in the risk-neutral measure), the credit spread increases as the credit quality worsens. Therefore we could use the given spread to determine the rating and hence the expected distribution for a particular issuer.

For example, consider an issue for which the credit spread is about z_t=36 basis points. Even though the issuer for the bond under consideration is rated A, its credit quality is in between ratings A and BBB. According to the figures in Table 3, we could place the rating of the issuer at about 60% A and 40% BBB. For this particular case the spread scenarios should be defined as

$$z_i = z_t + 60\%\ \xi_i(\mathrm{A}) + 40\%\ \xi_i(\mathrm{BBB})\,, \qquad (26)$$

where $\{\xi_i(\mathrm{A})\}$ and $\{\xi_i(\mathrm{BBB})\}$ are the additive scenario bases for the historically simulated z-spread respectively for the A and BBB ratings defined by (16). The expected distribution for the spread scenarios looks like the one shown in Figure 2 as a thin solid line.

To express these ideas in a precise mathematical form, consider an issuer with a certain rating, e.g. A, for which it is possible to bootstrap a default probability term structure. Consider then a bond with maturity date y_m, i.e. exactly the same maturity as the synthetic bond B^m for some m(one of the maturities considered in Table 2). As mentioned in subsection in 4.1, given the default probability structure for the issuer it is possible to compute the z-spread z_t^m associated with the bond B^m. Next, find the two *bracketing* ratings, e.g. A and BBB, such that the computed spread z_t^m falls between the spread of the two ratings computed for the same bond B^m using ratings A and BBB, i.e.,

$$z_t^m(\mathrm{A}) \le z_t^m < z_t^m(\mathrm{BBB})\,, \tag{27}$$

where, in this example, the bracketing ratings are A and BBB, and $z_t^m(\mathrm{A})$ and $z_t^m(\mathrm{BBB})$ are the current z-spread computed for A and BBB respectively. Define the fractional-rating coefficient β^m as

$$\beta^m = \frac{z_t^m(\mathrm{BBB}) - z_t^m}{z_t^m(\mathrm{BBB}) - z_t^m(\mathrm{A})}\,. \tag{28}$$

This percentage represents how far is the current spread z_t^m from the spread of the worst rating, in this example $z_t^m(\mathrm{BBB})$. If β^m is zero then the bond is 100% a BBB. If β^m is equal to one we are dealing with an issuer that is 100% A. In the general case the issue has a *fractional rating* that is for β^m units rated A and for $1-\beta^m$ units rated BBB.

The reference spread to be used in the price computation at time t, as given by equation (13), is

$$z_t = z_t^m\,, \tag{29}$$

and the scenarios are computed according to equation (14), with[6]

$$z_i = z_t + \beta^m\,\xi_i^m(\mathrm{A}) + (1-\beta^m)\,\xi_i^m(\mathrm{BBB})\,, \tag{30}$$

where $\xi_i^m(\mathrm{A})$ and $\xi_i^m(\mathrm{BBB})$ are defined as in (16) for the two ratings A and BBB in turn.

Note When $z_t < z_t^m(\mathrm{AAA})$, interpolation between ratings is not possible and scenarios should be computed from equations (17) and (16) with a rating of AAA.

Example Figure 2 shows the plot of the original spread distribution of ratings A, BBB, and for three different fractional ratings, 93% A+7% BBB, 60% A+40% BBB, and 20% A+80% BBB; for the *all-sectors/all-subsectors* issuers and currency EUR. Notice how the worsening of rating from A to BBB, implies an increase of the reference spread and a widening of the distribution. Also, notice how the synthetic distributions relative to the fractional-rating coefficients β^m= 0.93, 0.6, 0.2, have qualitatively the same features as the original distributions. In particular, all the synthetic distributions are leptokurtic, i.e. present fat tails.

4.3 Spread scenarios for assets with any maturity date

Consider now the computation of spread scenarios for a given instrument, with a generic maturity y, different from all the maturities y_m's. Suppose also that the issuer of this instrument belongs to a specific sector and subsector as those listed in Table 1. Finally, suppose that it is possible to build a term structure for the default probability. Given the maturity y of the product, let m be such that $y_m \le y < y_{m+1}$ and compute γ as defined by (20). Also build two synthetic fixed-rate coupon bonds B^m and B^{m+1} with maturities y_m and y_{m+1} as described in section 3. Then compute the

[6] Alternatively to the linear interpolation of equation (30), we could use more sophisticated interpolation methods. However, we postpone to a future work the discussion of results obtained with other interpolation methods

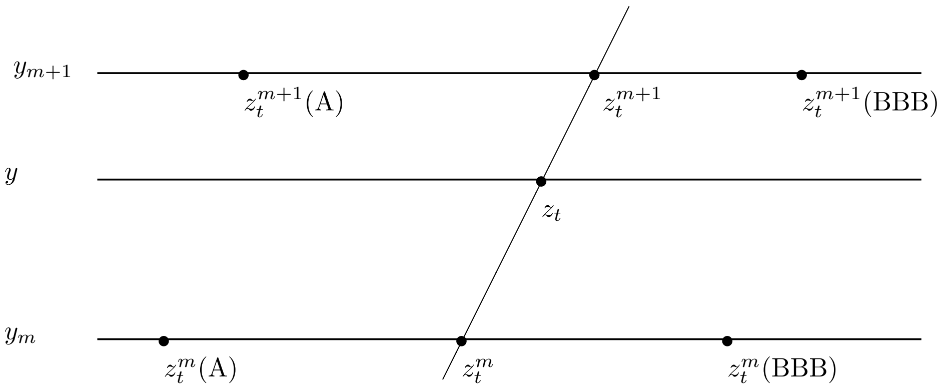

Figure 3: This Figure shows how the reference z-spread z_t is computed in the dynamic simulation method. The maturities are plotted as ordinates: y_m and y_{m+1} are the maturities of the two synthetic bonds and y is the maturity of the product under evaluation. The z-spreads z_t^m and z_t^{m+1} are those of the two synthetic bonds and z_t is computed from equation (33).

two reference z-spreads z_t^m and z_t^{m+1} associated with the two bonds B^m and B^{m+1}. At this point, similarly as it was shown in section 4.2, find the ratings, for example A and BBB, such that

$$z_t^m(\mathrm{A}) \le z_t^m < z_t^m(\mathrm{BBB})\,, \tag{31}$$

and the ratings, for example again(but they may be different from those in equation (31)), A and BBB , such that

$$z_t^{m+1}(\mathrm{A}) \le z_t^{m+1} < z_t^{m+1}(\mathrm{BBB})\,. \tag{32}$$

Define also the coefficients β^m and β^{m+1} as in (28).

The reference spread and scenarios should be built blending the reference and scenarios for the two maturities y_m and y_{m+1} as in subsection 3.3; therefore, the reference spread is defined as

$$z_t \;=\; \gamma\, z_t^m + (1-\gamma) z_t^{m+1}\,. \tag{33}$$

The simulated spreads z_i's should be a blend of the scenarios at the two different maturities and for the different ratings,

$$\begin{aligned} z_i = z_t + \gamma\, &[\beta^m\, \xi_i^m(\mathrm{A}) + (1-\beta^m)\, \xi_i^m(\mathrm{BBB})] + \\ &+ (1-\gamma)\left[\beta^{m+1}\, \xi_i^{m+1}(\mathrm{A}) + (1-\beta^{m+1})\, \xi_i^{m+1}(\mathrm{BBB})\right]\,. \end{aligned} \tag{34}$$

These two equations can be used together with expressions (13) and (14) to compute the scenario distribution of a generic instrument with credit risk.

Notes

- When the issuer rating is AAA the method described in section 3 is always used.
- As in the case of section 3, if y is such that $y < y_1$ or $y > y_{n_m}$, we still define β^m as in (28) using the first or the last maturity available.

5 Summary and conclusions

In the present paper we illustrated how to construct credit-risk scenarios in the StatPro simulation model. We have shown that, when applicable, the dynamic approach should be preferred as it can model credit risk following market expectations very closely. In the following, we review the different methods to compute credit risk compare their weaknesses and strengths.

In section 2, we showed how default risk at the current time t, is already accounted for by the instrument price. The current price indeed is given by the expected value, in the risk-neutral measure, of all the future cash flows, hence, this is the so-called **risk-neutral approach**. From the risk management point of view the computation of the current price is just the beginning of the process. Usually, we need a probability distribution in the real world measure for the price of the asset at the future time T. In general, the perception of credit risk at time T will be different and, therefore, the term structure of the default probability is different. As shown in subsection 2.2, the changes in credit worthiness can be summarized by changes in the zero-volatility credit spread.

Given an issuer belonging to a certain *sector/rating/currency* group, it is possible to derive from the asset-swap historical data an equivalent time series of credit spreads. In section 3 we showed how to use this spread series to generate credit scenarios for the current issue. As expected, see also Figure 1, the spread distribution becomes wider and wider as the rating decreases, i.e. deteriorates. This is the **static approach** because the width of the spread distribution does not vary once the issuer rating has been established and only varies because of a rating upgrade or downgrade. If, for any change of issuer rating, the spread series used to compute the spread scenarios is updated promptly, the spread-distribution width and shape vary in sync with the issue notices from the rating agencies.

In the **historical-simulation** approach, described in subsection 4.1, the historical spreads generated from the historical default probabilities are used directly to generate the spread scenarios. This approach, however, does not seem to respond quickly enough to credit events. Indeed, the simulated distribution presents the right location for its reference but not the correct shape.

In subsection 4.3 we showed how it is possible to follow market credit expectations for those issuers for which a daily term structure for the default probability is available. For these issuers it is possible to define each day a fractional rating and, hence, a different expected spread distribution. This is called the **dynamic approach** as it is not necessary to wait for a rating-agency update to change the spread distribution width and shape. The changes happen as a consequence of changing market expectations.

To conclude, the **risk-neutral** approach models very well the current state of credit risk but does not in any way model the changes in risk worthiness. It is not properly a risk-management approach but a pricing approach.

The **historical-simulation** approach has the advantage of capturing the idiosyncratic default risk since the current price is computed with the correct spread, but has two weaknesses:

- It cannot be used for issuers that do not readily have an individual default probability curve available.
- Event risk is captured, however, in a risk-function dependent way: i.e., different risk functions(VaR, expected shortfall, volatility) include the new scenarios in different time scales.

The **static** approach, has the advantage to be general enough to cover also *illiquid* issuers for which it is not possible to compute a default probability term structure. This approach also covers the *event risk* linked to upgrades or downgrades of the ratings, however, presents a few shortcomings:

- Idiosyncratic risk is not captured as price is computed using the average spread for the *sector/rating/currency* group.

- Two or more issuers in the same *sector/rating/currency* group will have the same spread scenarios and therefore their scenarios will be highly correlated.
- Those event risks not linked to ratings are not captured even when the credit merit of an issuer improves or deteriorates. The method ignores new credit condition that are not strong enough to trigger a rating change.

Finally, the **dynamic** approach has many of the advantages of the previous two approaches

- It captures idiosyncratic risk as the credit spread used in pricing is built from the specific risk-neutral default probability of the specific issuer at its maturity.
- The response to event risk is immediate as the approach reflects the risk that is embedded in the market-quoted default probability structure. From day to day, as the fractional rating changes, the spread distribution changes in shape and location.
- Unfortunately, this approach requires the availability of a credit-default swap curve for the issuer or some other means to bootstrap the default probability curve.

It is in the opinion of the authors that the static approach should be used for all the issuers for whom it is not possible to compute a default curve; the dynamic approach should be used for all other cases. The dynamic approach has also the advantage of combining event and default risk inside the StatPro simulation method, greatly improving the quality of the resulting risk management model.

Note The static and dynamic approaches described in this paper are implemented by StatPro and are available to clients using the StatPro Risk Management Product also known as *SRM* or *StatVar*.™

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